Roundtable on Coherence

Introductory remarks

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Credences

- Branden dealt with **full beliefs** and **full disbeliefs**.
- I will deal with **partial beliefs** or **credences**.
- Represent an agent by her credence function c:
  - $c(A)$ is a real number in $[0, 1]$.
  - It measures her credence in A.

Coherence for credences

Coherence principles say how credences in propositions with a particular logical form relate to propositions with a related logical form.

E.g.

- **Probabilism**
  1. $p(\text{Contradiction}) = 0$ and $p(\text{Tautology}) = 1$.
  2. If A and B are mutually exclusive, $p(A \vee B) = p(A) + p(B)$.
- **Principal Principle** $p(A \mid \text{The chance of } A \text{ is } x) = x$.

We'll be concerned with **Probabilism** and, to a much lesser extent, the **Principal Principle**.

The three-step strategy in the case of full belief

- **Step 1:** To each world w, assign a set of beliefs/disbeliefs $\mathbf{B}_w$ that is **vindicated** at that world.
- **Step 2:** For any set of beliefs/disbeliefs $\mathbf{B}$ and any world w, define a **measure of distance** from $\mathbf{B}$ to $\mathbf{B}_w$.
- **Step 3:** Choose a **fundamental principle**.

The three-step strategy in the case of credences

- **Step 1:** To each world w, assign a credence function c_w that is **vindicated** at that world.
- **Step 2:** For any credence function c and any world w, define a **measure of distance** from c to c_w.
- **Step 3:** Choose a **fundamental principle**.

The vindicated credence function

Suppose w is a world. What is c_w?

Joyce's answer:

$$c_w(A) = \begin{cases} 1 & \text{if } A \text{ is true} \\ 0 & \text{if } A \text{ is false} \end{cases}$$

Again, c_w is the cognitive state of an omniscient agent:

- Maximal credence in truths;
- Minimal credence in falsehoods.

The measure of distance

Suppose w is a world and c_w is as defined above.

There are many putative measures of distance d from c to c_w:

- Joycean inaccuracy measures.
- Proper scoring rules.

The following is in both sets:

$$d(c, c_w) = \sum_{A} (c(A) - c_w(A))^2$$

It is called the *Brier score*.

The fundamental principle

Weak Accuracy-Dominance Avoidance (WADA$_d$)

c is not weakly dominated.

That is, there does *not* exist an alternative credence function c' such that

(i) $(\forall w)[d(c', c_w) \leq d(c, c_w)]$;

(ii) $(\exists w)[d(c', c_w) < d(c, c_w)]$.

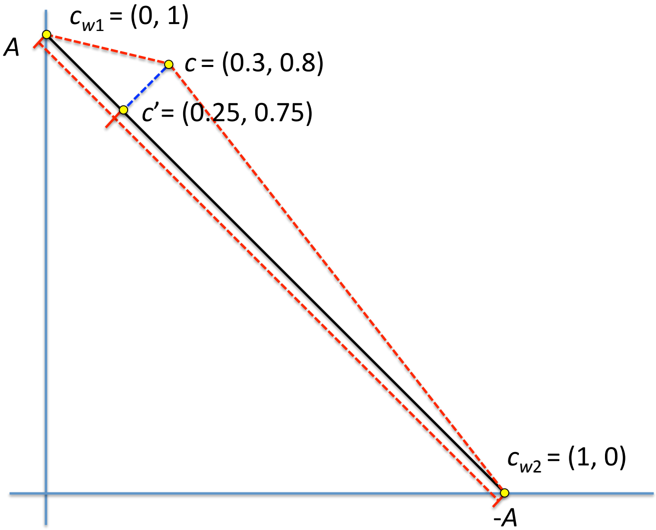
The consequences

Theorem 1 (de Finetti 1974, Joyce 1998, Predd et al 2009)

Suppose d is a measure of distance from c to c_w. Then

$$c \text{ satisfies } (\text{WADA}_d) \quad \Leftrightarrow \quad c \text{ satisfies Probabilism.}$$

The theorem illustrated



Another fundamental principle

Weak Accuracy Chance Dominance Avoidance (WACDA_d)

c is not chance dominated.

That is, there does *not* exist an alternative credence function c' such that

$$\text{Exp}_{ch}(d(c', c_w)) < \text{Exp}_{ch}(d(c, c_w))$$

for all chance distributions ch.

The consequences

Theorem 2

Suppose d is a measure of distance from c to c_w. Then

c satisfies (WACDA$_d$) $\Leftrightarrow$ c satisfies Prob + Principal Principle.

References

My notes for the DGL tutorial on this topic are available at: https://dl.dropbox.com/u/9797023/Talks/EUT.pdf.

- de Finetti, Bruno (1974) *Theory of probability* Vol. 1 (New York: Wiley).
- Joyce, James M. (1998) 'A Non-Pragmatic Vindication of Probabilism' *Philosophy of Science* 65: 575-603.
- Predd, Joel, Robert Seiringer, Elliot H. Lieb, Daniel N. Osherson, H. Vincent Poor, and Sanjeev R. Kulkarni (2009) 'Probabilistic Coherence and Proper Scoring Rules' *IEEE Transactions on Information Theory* 55(10): 4786–4792.