Fairness and Jutsified Representation in Judgment Aggregation and Belief Merging

Proportional fairness of a voting rule can be characterized as the ability to reflect all shades of political opinion of a society within the winning committee. The purpose of our project is to apply the voting theory tools concerning proportionality with respect to the rules which take approval ballots as input to the framework of judgment aggregation. Recently Aziz et al. and Brill et al defined certain two proportionality properties, called justified representation and extended justified representation. A rule satisfies extended justified representation (EJR) if for each approval election with n voters, each committee size k, and each $l <= k$, the following holds: There is no group of $l \cdot \frac{n}{k}$ voters that all approve at least l common candidates, but neither of whom approves l or more members of each winning committee. A rule satisfies justified representation (JR) if it satisfies EJR for $l = 1$.

To be precise:

Definition 1 (Justified Representation in the Context of Judgment Aggregation)**.** *Let N be the of judges of size n, and let Φ be their agenda. Suppose f is an aggregation rule for N and Φ. Let $\mathbb{J}$ be a profile. We say that a set of formulas $\Psi \subseteq \Phi$ of size k* ***provides justified representation*** *for $(\mathbb{J}, k)$ iff*

$$\forall A \subseteq N \ \left(|A| \geqslant \frac{n}{k} \ \wedge \ \exists \varphi \in \Phi \forall i \in A \ \varphi \in J_i\right) \Rightarrow (\exists i \in A \exists \psi \in J_i \ \psi \in \Psi),$$

or, in more familiar terms:

$$\forall A \subseteq N \ \left(|A| \geqslant \frac{n}{k} \ \wedge \ \bigcap_{i \in A} J_i \neq \emptyset\right) \Rightarrow \left(\bigcup_{i \in A} (J_i \cap \Psi) \neq \emptyset\right).$$

We say that the aggregation rule f ***satisfies justified representation*** *(JR) if for every profile $\mathbb{J} \in \mathcal{J}(\Phi)^n$ and every target size k of a set of accepted propositions it holds that $f(\mathbb{J})$ provides justified representation for $(\mathbb{J}, k)$.*

Intuitively, justified representation requires that, if there is a group of at least $\frac{n}{k}$ voters whose approval ballots have at least one candidate in common, then it cannot be the case that neither of these voters is represented in the committee. EJR extends this reasoning to larger groups of voters and larger sets of jointly approved candidates. Aziz et al. showed that PAV is the only approval-based rule which satisfies EJR. Brill et al., on the other hand, discussed a relation between multiwinner voting rules and methods of apportionment, which allows to view PAV as an extension of the d'Hondt method of apportionment to the multiwinner setting. The question of transferring these results into judgment aggregation setting becomes interesting when one acknowledges that voting theory studies the aggregation of individual preferences simpliciter, while the theory of judgment aggregation investigates how individual opinions on logically related propositions can be consistently aggregated into a collective position with the consistency requirement providing quite a strong constraint for the rule. Simultaneously, judgmenet aggregation can be seen as a generalization of the election rules. During the talk I will try to demonstrate how we can use the machinery from the field of multwinner election theory to investigate proportionality properties in the general situation of voting for logical propsitions (thus, related logically) in place of candidates only, prove some indepencende results, and focus on a recently re-deiscovered voting rule of Phragmen. If time permits, I will also try to report on how this relates to some current research in the area of belief merging, and generalize the framework to the probabilistic version. This is a joint work-in-progress with Piotr Skowron.

Due to lack of space I do not announce any of the relevant formal results (or - apart from JR - definitions).