Imprecise credences can increase accuracy wrt. claims about expected frequencies

Let's take credal states to be probabilistic measures or non-empty sets thereof. An agent is a *preciser* if their credal state is captured by a single probabilistic measure, and they are an *impreciser* otherwise. Consider the following scenario:

Endpoints mystery coin (EMC)	The opponent will produce two coins with the objective chances of Heads .3 and .5, randomly pick one of these coins and then toss it.

and suppose the only propositions that we care about are that the result will be heads (H) and that the result will be tails (T). One epistemic strategy that might seem natural is to form an imprecise credal state comprising two precise ones c_1 and c_2, such that $c_1(H) = .3, c_1(T) = .7, c_2(H) = .5$ and $c_2(T) = .5$. Another option, however, is to accept a precise credal state c with $c(H) = .4$ and $c(T) = .6$. The question arises: are there any compelling rationality constraints that would require the agent to choose one of these options? Aren't these responses on a par in terms of their rationality?

You might think that the imprecise strategy should come recommended, because, in some sense, it is responsive to the evidence in a way that the precise strategy isn't. You migh further have the inuition that had we been dealing simply with one toss of a coin with bias .4, it would be more appropriate to form c rather than $\{c_1, c_2\}$. However, it seems that it is hard to reconcile this intuition with epistemic utility theory, according to which what ultimately matters in credal state evaluation is accuracy. Schoenfield (2017) developed a fairly general argument regarding such issues. It pertains to cases that involve two propositions (heads/tails, H/T). Let $\mathcal{X}$ be a partition of the sample space. A consistent truth-value (TV) assignment into $\{0, 1\}$ yields 1 for exactly one member of $\mathcal{X}$, X_i. All such TV assignments are denoted as $V_\mathcal{X}$. Denote the set of all probabilistic precise credence functions from $\mathcal{X}$ into $[0, 1]$ by $C_\mathcal{X}$. Imprecise credal states are taken to be sets of credences over $\mathcal{X}$. Denote all such sets as $B_\mathcal{X}$. Now consider accuracy measures, one for precise credences, and one for imprecise credences, so that $\mathcal{G}_\mathcal{X} : C_\mathcal{X} \times V_\mathcal{X} \mapsto [0, 1]$ and $\mathcal{G}^\star_\mathcal{X} : B_\mathcal{X} \times V_\mathcal{X} \mapsto [0, 1]$, such that $\mathcal{G}^\star_\mathcal{X}$ is an extension of a plausible measure $\mathcal{G}_\mathcal{X}$, so that $\mathcal{G}^\star_\mathcal{X}$ gives 1 to any c that gives 1 to all truths and 0 to all falsehoods, and $\mathcal{G}^\star_\mathcal{X}$ gives 0 to any c that yields 0 to all truths and 1 to all falsehoods. $\mathcal{G}_\mathcal{X}$ is assumed to be continuous. Suppose $\mathcal{G}^\star_\mathcal{X}$ is bounded by 0 and 1, (sometimes) takes these values, and that for any partition $\mathcal{X}$ and any probabilistic belief state b over $\mathcal{X}$ there is no belief state b' over $\mathcal{X}$ more accurate for some X_i and no less accurate for all X_i. Given these assumptions, Schoenfield shows, for any probabilistic imprecise i over H/T and any $\mathcal{G^\star}_{H/T}$, there is a precise probabilistic p no less accurate than i for any $v \in V_{H/T}$. The philosophical lesson from this is supposed to be that by Schoenfield's theorem, for any imprecise b over H/T there is a precise b' no less accurate than b, and so, arguably, there can be no rational requirement to adopt an imprecise b over H/T.

Suppose, indeed, in a particular case in which the agent's partition is fairly limited (H/T), as far as accuracy is concerned, no imprecise credal state beats all precise credences, and so no imprecise credal state is recommended as such. Does it follow that in somewhat more complex circumstances in which we consider other propositions, the theorem applies? I will argue that even if we restrict our attention to a fairly natural class of a finite number of propositions whose truth depends on the results of coin tosses and the bias of the coin that's being tossed, the preciser will end up less accurate than the impreciser.

Here is what I take to be a fairly sensible assumption in the contexts in which it makes sense to talk about objective chance and repeated Bernoulli trials with the same probability of success.

Negative Frequency Criterion (NFC)	If an agent's relevant evidence regarding the results of a finite number of tosses of one and the same coin does not change, then if the agent believes that as the number of tosses increases, the frequency of Heads will not tend toward L, then for no coin toss i and for no probabilistic measure c in the agent's credal state, it is rational to have $c(H_i) = L$.

The idea seems simple enough: our well-informed expectations of frequencies in repeated Bernoulli trials should guide our credences in particular outcomes of these trials, at least in the negative sense. If we think the frequency of Heads will not tend to L, and we have no particular reasons to think that coin toss i will be somehow special, we should not have $c(H_i) = L$ in our (precise or imprecise) credal state.

The plan is to:

- Argue that once the agent is allowed to have credal states towards propositions about limiting frequencies, (NFC) will commit the preciser to highly improbable claims to which the impreciser will not be committed.
- Defend (NFC) by considerations related to the so-called *statistical syllogism* (Taroni et al., 2006, 20).
- Defend (NFC) by considerations related to the fact that if you're a preciser, your expected frequency should be the same as your credence in each particular H_i.

Thus, while Schoenfield's theorem holds, it only shows that the impreciser has no advantage over the preciser over an aritficially narrow algebra of propositions, and my arguments show that once you realistically extend considerations to credal states about claims involving frequencies, the theorem no longer applies, and there are clear cases in which the preciser ends up less accurate.

References

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