Mathematical Philosophy

LMU Munich

Hannes Leitgeb

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Mathematical Philosophy—the application of logical and mathematical methods in philosophy—is experiencing a tremendous boom in various areas of philosophy. This Alexander von Humboldt Professorship project founds a new Center for Mathematical Philosophy at the Ludwig-Maximilians-University Munich at which philosophical research is carried out mathematically, that is, by means of methods that are very close to those used by the scientists. The purpose of doing philosophy in this way is not to reduce philosophy to mathematics or to natural science in any sense; rather mathematics is applied in order to derive philosophical conclusions from philosophical assumptions, just as in physics mathematical methods are used to derive physical predictions from physical laws. Nor is the idea of mathematical philosophy to dismiss some of the ancient questions of philosophy as irrelevant or senseless: although modern mathematical philosophy owes a lot to the heritage of the Vienna and Berlin Circles of Logical Empiricism, unlike the Logical Empiricists most mathematical philosophers today are driven by the same traditional questions about truth, knowledge, rationality, the nature of objects, morality, and the like, which were driving the classical philosophers, and no area of traditional philosophy is taken to be intrinsically misguided or confused anymore. It is just that some of the traditional questions of philosophy can be made much clearer and much more precise in logical-mathematical terms, for some of these questions answers can be given by means of mathematical proofs or models, and on this basis new and more concrete philosophical questions emerge. This may then lead to philosophical progress, and ultimately that is the goal of this Alexander von Humboldt Professorship project on mathematical philosophy.

Some of Our Research Topics:

Apart from an emphasis on logical-mathematical methods, research at the Center is completely free to be originated by its fellows, and it is open to stimulation by visitors. What holds the academic activities of Center together is thus a methodological theme, but even this theme will comprise a *plurality of methods*: from classical first-order logic over classical proof theory and non-classical logic to modal logic, from model theory over possible worlds semantics to the semantics of dynamic epistemic logic, from standard probability theory to non-standard accounts of degrees of belief, from graph theory to game theory, from Carnapian explications of concepts in the style of mathematicians to more heuristical model-building in the style of the natural scientists; and so forth. Hence, the following list of research topics and questions is deliberately tentative, even though it is likely that most, if not all, of these subjects will be in the focus of some members of our Munich Center for Mathematical Philosophy at some point of time. Furthermore, many of the following research questions overlap with each other, so that pursuing various of them simultaneously should have very positive synergetic effects:

- *Formal theories of truth and modality, and the semantic paradoxes:* Since Alfred Tarski's famous work on truth in the 1930s and 1940s, and since Saul Kripke's new take on the Liar paradox in the 1970s, the semantics of the truth predicate and the axiomatization of theories of truth has been a major research topic in philosophical logic and philosophy of language. Recently, Hartry Field's monograph and articles on truth and paradox caused a lot of new excitement about the subject. One goal of our research in the Center will be to find the answers to questions that might lead this area into some new and fresh directions, such as: Is there an inferentialist justification of recent theories of truth? Does the truth predicate have explanatory power after all, *contra* deflationism about truth? Is there a joint axiomatic theory of propositions and truth in which semantical paradoxes are avoided in ways that are similar to the manner in which modern axiomatic set theory avoids the set-theoretic paradoxes? Is there a consistent, systematic, and philosophically illuminating way of expressing modalities such as necessity or knowledge in terms of predicates of sentences, and how can the notorious paradoxes that have been claimed to affect such a treatment of modalities be avoided? Accordingly: Assume the truth predicate to be replaced by a probability sign for sentences: what does a semantically closed theory of probability look like in which probabilities are assigned even to sentences that speak about their own probability?

- *Inferentialism about meaning*: According to inferentialists about meaning, the meaning of linguistic items is not so much given by the truth conditions for the sentences in which these linguistic items occur, but rather by the rules of inferences in which these linguistic items figure. Recently, the logical constraints have come under close scrutiny that make the constitution of meaning in this sense possible in the first place: for if rules of inference do not satisfy these constraints, then non-intended or even contradictory inferential practices may emerge. Most importantly, so-called principles of harmony have been formulated which are meant to establish a balance between rules of introduction and rules of elimination for linguistic items. But what is the correct formal formulation of such principles of harmony? How does harmony relate to well-known truth-theoretic proposals of how the semantic paradoxes are to be avoided (such as principles of groundedness for truth, contextual relativization of quantifiers, and the like)? How plausible is inferentialism about logical constants from the viewpoint of empirical work on our understanding of, and reasoning with, logical signs? Which additional assumptions have to be made in order to determine classical truth conditions from inferentialist accounts of meaning, if possible at all? Are there two kinds of meaning—one semantic, the other one pragmatic-epistemic—that are underlying the differences between truth-conditional and inferentialist theories of meaning? Are suppositional theories of conditionals, according to which a conditional is acceptable to an agent if and only the assumption of its antecedent allows for a rational reasoning process that ends with the acceptance of its consequent, inferentialist in the sense above?

- *Bayesianism—application, justification, and limitations thereof:* Since Rudolf Carnap's investigations into the probabilistic foundations of inductive logic, probability theory has become the dominant tool in formal epistemology and general philosophy of science. Unlike Carnap, modern day Bayesianists do not regard probabilities as determined by logical means only, but these probabilities are really subjective degrees of belief which are subject to certain rationality constraints. A substantial part of the research in the Center will be devoted to the application of probabilistic methods in this Bayesianist sense: Given a subjective probability measure, is there a uniquely determined measure of the degree of confirmation of a scientific hypothesis in the light of evidence that can be defined in terms of this probability measure? How can the degree of acceptability of indicative or subjunctive conditionals in natural language be measured probabilistically? What is the empirical content of a scientist's belief system if the scientist's beliefs are given in terms of probabilistic degrees of belief? We will not just investigate questions like these by mathematical means, but we also plan on testing some of our hypotheses by means of experiments with actual subjects in the style of what is called *experimental philosophy* these days; at the same time we want to get a clearer picture of the possible limitations of such applications of empirical methods in philosophy. Finally, we will be interested in the justification of Bayesianism: Is it possible to derive the axioms of probability theory from more fundamental epistemic maxims, such as: always try to stay as close as possible to the truth? What additional assumptions have to be made in order to justify distributing probabilities uniformly in some domain of reasoning, and is it possible to learn on the basis of uniform probabilities despite the classical Carnapian worries about this? For which questions of general philosophy of science are the current Bayesianist approaches too restrictive? For instance: Are there concrete examples from scientific practice in which the confirmation of a hypothesis H in the light of evidence E and given some background theory K depended on more than just absolute or conditional probabilities involving H, E, and K?

- *Conditionals, supposition, and the logic of conditionals:* Conditionals, that is, if-then sentences, play a major role in everyday language, science, and in philosophy itself: in natural language discourse we communicate regularly in terms of indicative or subjunctive conditionals; natural scientists formulate law-like hypotheses in an if-then form; scientists in the life scientists do so as well but they take their if-then "laws" to allow for exceptions; computer scientists have been developing a whole new area of research that deals with if-then instructions that are meant to be applicable only in the normal or default case; and in metaphysics the standard philosophical theories of laws, dispositions, and causality all involve conditional idioms. In the Center we will investigate the logic, semantics, pragmatics, and epistemics of conditionals, we will apply our findings in various of the different areas and contexts mentioned before, and we will do so by using different mathematical methods—in particular, logical and probabilistic ones—and experimental methods. For instance: How does reasoning in terms of conditionals relate to suppositional reasoning? Is the famous Ramsey test for conditionals empirically confirmed? Which logical rules for conditionals do competent speakers obey in natural language? Do David Lewis' triviality results show that indicative conditionals are neither true nor false and what would this mean for realist conceptions of scientific theories that include if-then statements? What is the proper formulation of the Ramsey test for subjunctive conditionals? Do the degrees of acceptability for indicative and subjunctive conditionals differ even in cases where these conditionals talk about future events? Do counterfactuals have two kinds of meaning (one suppositional, one truth-conditional) which correspond to two kinds of degrees of acceptability? Are the most recent semantics for counterfactuals—similarity semantics, causal-structural equations semantics, conditional chance semantics—mutually translatable into each other? Is there a way of saving the traditional conditional analysis of dispositions by changing the conditional semantics of disposition terms?

- *Qualitative and quantitative belief:* There are (at least) two concepts of belief that get used in everyday language, science, and philosophy: One is qualitative, according to which one either believes that A is the case, or one disbelieves that A is the case, that is, one believes that not-A is the case, or one is agnostic about the whole alternative. The other one is quantitative, according to which one believes A to some numerical degree of belief that measures how firmly one believes in the truth of A. These two concepts are subject to different sets of rationality standards: qualitative belief has to meet logical requirements if it is to count as rational, while degrees of belief must satisfy the axioms of probability theory or otherwise it is rationally incoherent. None of the two concepts ought to be given up—for example, in epistemology and for the rational reconstruction of science we need both of them—but it would also be unacceptable to live with our postulates of rationality bifurcating into two mutually incomprehensible directions. So how can these two standards of rationality be unified in one joint theory? Is it possible to give an explicit definition of belief in terms of subjective probability, such that it is neither the case that belief is stripped of any of its usual logical properties, nor is it the case that believed propositions are trivially bound to have probability 1? And, on related grounds: How can the Bayesian approach to general philosophy of science be reconciled with the deductive or semantic conception of scientific theories and theory change? How can the assertability of conditionals become an all-or-nothing affair in the face of non-trivial subjective conditional probabilities? Does knowledge entail a high degree of belief but not necessarily certainty? Can high conditional chances become the truth-makers of counterfactuals? Can the approximate truth of propositions be defined on the basis of a semantic notion of probability?

- *The logic of action, reasons, preferences, and deontic logic:*

  For areas such as ethics and the philosophy of mind, the analysis of intentional actions, the reasons for actions, and the preferences between actions is indispensable. If someone is to count as a fully rational person, his or her actions must be committed intentionally, there must be good reasons for the action at least from the viewpoint of that person, and among the possible actions from which the person could choose, the chosen action must be the most preferred one. All of these ingredients have a formal structure: next to the traditional logics for action there are also recent dynamic logics of action and intention; recently the logic of the rational change of preferences has become an important research topic of its own; social choice theory tries to capture the ways in which individual preferences can be aggregated into one social preference ordering in rationally permissible ways; the emergence of social norms and a social contract has been studied with the help of game theoretic methods; the logic and semantics of conditional norms has been developed in ways that are similar to the modern analyses of descriptive conditionals in natural language; and so on. Indeed, the application of logical and mathematical methods to the analysis of ethical concepts and theories is now a rapidly growing field of research. Research in the Center on this set of topics will address questions such as: Is it possible to capture rational decision-making completely in terms of classical decision theory, and if not what are the limitations of decision theory? How appropriate is the application of evolutionary game theory in the context of rational decision making? How do the well-known impossibility theorems on social choice, which concern the social aggregation of individual preferences, relate to well-known impossibility theorems on belief revision in which worldy preferences that determine the semantics of conditionals cannot be translated in any obvious manner into epistemic preferences that determine a rational agent's belief revision dispositions? Should the modal axioms for actions be formulated in terms of sentential operators or predicates or both? Is there a suppositional semantics of conditional norms that would be acceptable both to cognitivists and non-cognitivists about norms? If so: is such a semantics dynamic in nature, so that the if-part of a conditional norm leads to a change of one's moral preferences, and should such a change be construed in qualitative or in quantitative terms?

- *Structuralism about mathematics, scientific progress, and truth:* Structuralists about mathematics argue that mathematical individuals, such as natural numbers, real numbers, or sets, are ontologically secondary to the structures that they occupy, such as the natural number structure, the real number line, or the cumulative hierarchy of sets. The "real" objects of mathematics are structures, that is. Similarly, structural realists about scientific progress maintain that what is preserved when one empirically successful scientific theory is replaced by an empirically even more successful theory is the "structural content" of the original theory; structurally, what was successful about the old theory, is in fact preserved in the course of transition. While there has been a lot of more or less informal philosophical debate on both mathematical structuralism and structural realism, there is neither an axiomatic treatment of structuralism about mathematics by which it would become clear how, e.g., natural numbers as positions-in-a-structure differ from, say, natural numbers reconstructed set theoretically, nor is there a formal explication of what the structural content of an empirically successful theory might be like. Previous proposals for the former in terms of quasi-set-theoretic foundations of mathematical structuralism have proved equally unsatisfying as previous proposals for the latter in terms of defining the structural claims of a theory by means of the so-called Ramsification of theoretical terms. In the Center we intend to supply structuralists in either sense with formalized ways of making their claims more precise; if this turns out to be impossible, we want to translate the corresponding findings into arguments against such structuralist approaches. For instance: Is there an axiomatic theory of unlabelled graphs in which graphs are not ordered pairs of a set of vertices and a set of edges but according to which unlabelled graphs are really structures in a serious sense of the word? If so, is it possible to generalize such a theory to a foundational theory for structures in general? Does the actual practice of graph theorists lend support to such a structuralist account of graphs? Does the logical reconstruction of theoretical terms in scientific theories by means of Hilbert's epsilon terms, as suggested by Carnap in the 1950s and 1960s, allow for a structuralist interpretation of scientific theories? And if so, what do the well-known proof-theoretic results about epsilon terms then tell us about the structural features of scientific theories? Finally: Is it possible to interpret the concept of truth in similarly structuralist terms, such that the truth concept would pick out a structural property in a similar sense as—if structuralists about mathematics are right—the successor concept for natural numbers picks out a structural relation?

- *Inductive logic and neural networks:* The dynamical-connectionist paradigm in cognitive science, according to which cognitive systems are to be understood as neural networks, has traditionally been opposed to logical accounts of cognition. In the meantime, with the introduction of new logical formalisms, the role of logic for our understanding of neural networks has changed crucially: various of the new logical systems are provably sound and complete with respect to a semantics by which possible worlds or environmental states are ordered according to their degree of normality or typicality and in which logical operators are used to express properties of worlds or states with minimal degree of abnormality; by mapping such formulas to distributed representations in networks, these normality orderings can be put into correspondence with the distribution of energy over network states, such that maximally normal worlds reflect preferred network states of minimal energy in neural networks; inference or revision on the basis of formulas thus matches the convergence of net states towards equilibria given input. Another aim of the research in the Center is to take this logical approach to reasoning in neural networks one step further: Is it possible to translate the new findings on logical descriptions of the dynamics of artificial neural networks into a logical-cognitive account of learning in such neural networks? That is: are there logical representations of the transitions from one assignment of weights to the edges of a neural network to another such assignment on the basis of a learning algorithm and a set of training data? Do different learning schemes such as Hebbian learning or backpropagation correspond to different sets of rules of inductive logic for conditionals and singular data? How can probability measures be represented in artificial neural networks, such that the propositions to which probabilities are assigned, as well as the degrees of belief that are assigned to them, are represented in the distributed fashion that is characteristic of the connectionist paradigm in cognitive science? We will investigate questions like these not only by means of mathematical methods, but computer simulations will be equally crucial both to visualize the processes in question and to test the hypotheses about the logical accounts of learning in networks that we hope will emerge. In this way, a whole new naturalistic foundation of inductive logic could be in the making.

- And many more . . .

Research topics and questions such as these are not only pursued by the institutional members of the Center themselves, but we also attract visitors—philosophers and scientists—to come to the Center, to spend some time here, and to work on these topics collaboratively.

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