A New Lottery Paradox for Counterfactuals

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Once a week, a TV lottery takes place which is hosted by a famous entertainer. One day the host has a serious car accident on his way to the studio; out of respect for his condition, the lottery show is being cancelled. At the end of the day, the situation is fairly summarized by our first premise P1.

P1 If *A* had been the case, *B* would have been the case.

("If the host had made it to the studio, there would have been the TV lottery that day.")

It could have been the case that the host would not have had the accident and hence would have made it to the studio.

And it happens to be the case that the TV lottery is a lottery with 1.000.000 tickets; assume that it would not be the TV lottery anymore if this were not so:

P2 A is possible; and necessarily: B if and only if $C_1 \vee \ldots \vee C_{1000000}$.

("The host could have made it to the studio; and necessarily: the TV lottery would have taken place that day if and only if ticket 1 or ticket 2 or ... or ticket 1.000.000 would have won in the TV lottery that day.")

The set of true counterfactuals is of course closed under all logical rules and includes all logical laws.

We suppose the system V of conditional logic, which is a subsystem of David Lewis' (1973) preferred logic VC (= V + Centering Axioms), to be valid:

P3 All axioms and rules of the system V of conditional logic are valid.

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In particular:

$$\text{Agglomeration:} \quad \frac{\varphi \mathrel{\Box\!\!\rightarrow} \psi,\ \varphi \mathrel{\Box\!\!\rightarrow} \rho}{\varphi \mathrel{\Box\!\!\rightarrow} \psi \wedge \rho}$$

$$\text{Rational Monotonicity:} \quad \frac{\varphi \mathrel{\Box\!\!\rightarrow} \rho,\ \neg(\varphi \mathrel{\Box\!\!\rightarrow} \neg\psi)}{\varphi \wedge \psi \mathrel{\Box\!\!\rightarrow} \rho}$$

If a counterfactual is true—if φ had been the case, ψ would have been the case—then it is plausible to assume that its consequent ψ should have had a greater chance to have been the case than its negation $\neg\psi$, conditional on the antecedent φ:

P4 If a counterfactual of the form $\ulcorner$if φ then $\psi\urcorner$ is true, then the conditional chance of ψ given φ is greater than $\frac{1}{2}$.

In fact, in many cases, it should be possible to strengthen P4 by replacing '$\frac{1}{2}$' by some threshold closer to 1 that would be given contextually in some way.

If so, P4 above is really not more than just a minimal requirement.

Assume that the host had made it to the studio. Even then there would have been a small chance for the lottery being cancelled.

We assume the chance for the cancellation to happen was small but not tiny; indeed, we suppose that the chance of the lottery not taking place given the host had made it to the studio is bounded from below by the (tiny) chance of any particular ticket i winning in this lottery of 1.000.000 tickets:

P5 For all i: The conditional chance of $A \wedge C_i$ given $(A \wedge C_i) \vee (A \wedge \neg B)$ is less than, or equal to, $\frac{1}{2}$.

(“For all i: The chance of the host making it to the studio and ticket i winning given that either the host had made it to the studio and ticket i had won or the host had made it to the studio and the lottery had not taken place, is less than, or equal to, one-half.”)

As things stand, each of these premises is plausible if considered just by itself.

However, one can show that all of the premises P1–P5 taken together logically imply a *contradiction!*

Which premise is to go?

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None!?

My solution: *Contextualism about what counts as a proposition.*

- The context of assertion determines the space of propositions and in this way also which sentence expresses a proposition and which does not.
- Each of the premises before is saved in at least some context.
- But in no context all of them are satisfied simultaneously.
- Logical (P3) and "quasi-logical" premises (P4) are indeed satisfied in all contexts.

We consider two contexts c and c':

- Let the algebra $\mathfrak{A}_c$ be generated from

$$\{@\}, \{\underbrace{w_1, \ldots, w_{1000000}}_{u}\}, \{w^*\}$$

This yields $8 = 2^3$ propositions.

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- Let the algebra $\mathfrak{A}_{c'}$ be

$$\wp(\{@, w_1, \ldots, w_{1000000}, w^*\})$$

So $\mathfrak{A}_{c'}$ includes $2^{1000002}$ propositions.

Then we distribute chances:

- $Ch(\{@\}) = \frac{4}{7}$
- $Ch(\{w_1\}) = \ldots = Ch(\{w_{1000000}\}) = \frac{2/7}{1000000}$
  $Ch(\{u\}) = Ch(\{w_1, \ldots, w_{1000000}\}) = \frac{2}{7}$
- $Ch(\{w^*\}) = \frac{1}{7}$

Next we determine sphere systems $\mathfrak{S}_c$ and $\mathfrak{S}_{c'}$; it will be sufficient to determine the similarity orderings $\leq_c^@$ and $\leq_{c'}^@$ only for the actual world @:

- In the case of c, let

$$@ <_c^@ u <_c^@ w^*$$

- And for c', let

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$\leq_c^{@}$ and $\leq_{c'}^{@}$ satisfy this joint condition on $<$ (defined from $\leq$) and chance:

For all w: the chance of $\{w\}$ is greater than the sum of chances of sets $\{w'\}$ for which $w < w'$.

Finally, let an *expressing* relation be determined compositionally and relative to contexts, so that

- A expresses $\{w_1, \ldots, w_{1000000}, w^*\}$ both in c and c',
- B expresses $\{w_1, \ldots, w_{1000000}\}$ both in c and c',
- Each C_i expresses the proposition $\{w_i\}$ in c', whilst C_i does not express a proposition in c at all.

Accordingly, define truth for sentences relative to contexts.

With all the details being supplied, it follows:

Ad c: All premises of our initial argument are true in c except for

$$\Box(B \leftrightarrow C_1 \vee \ldots \vee C_{1000000})$$

which does not express a proposition in c (i.e., is not entertainable in c).

With all the details being supplied, it follows:

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which does not express a proposition in c (i.e., is not entertainable in c).

Ad c': All premises of our initial argument are true in c' except for

$$A \Box\!\!\rightarrow B$$

which is false in c'.

In order for P1 to hold in c' (given P4), it would be necessary that the chance of each proposition $\{w_i\}$ were greater than $\{w^*\}$. In other words: the chance of B given A would have to be much closer to 1.

Remark:

This is all orthogonal to Hanti & Kevin's (great!) results on partitions.

In their results they presuppose conditional acceptance to be a function of subjective probability.

But in the present context I do *not* suppose that the truth of counterfactuals is determined by chance—doing so would mean to beg the question against many philosophers in this area (e.g., Tim Williamson).