The Consistent Boundary Formulation

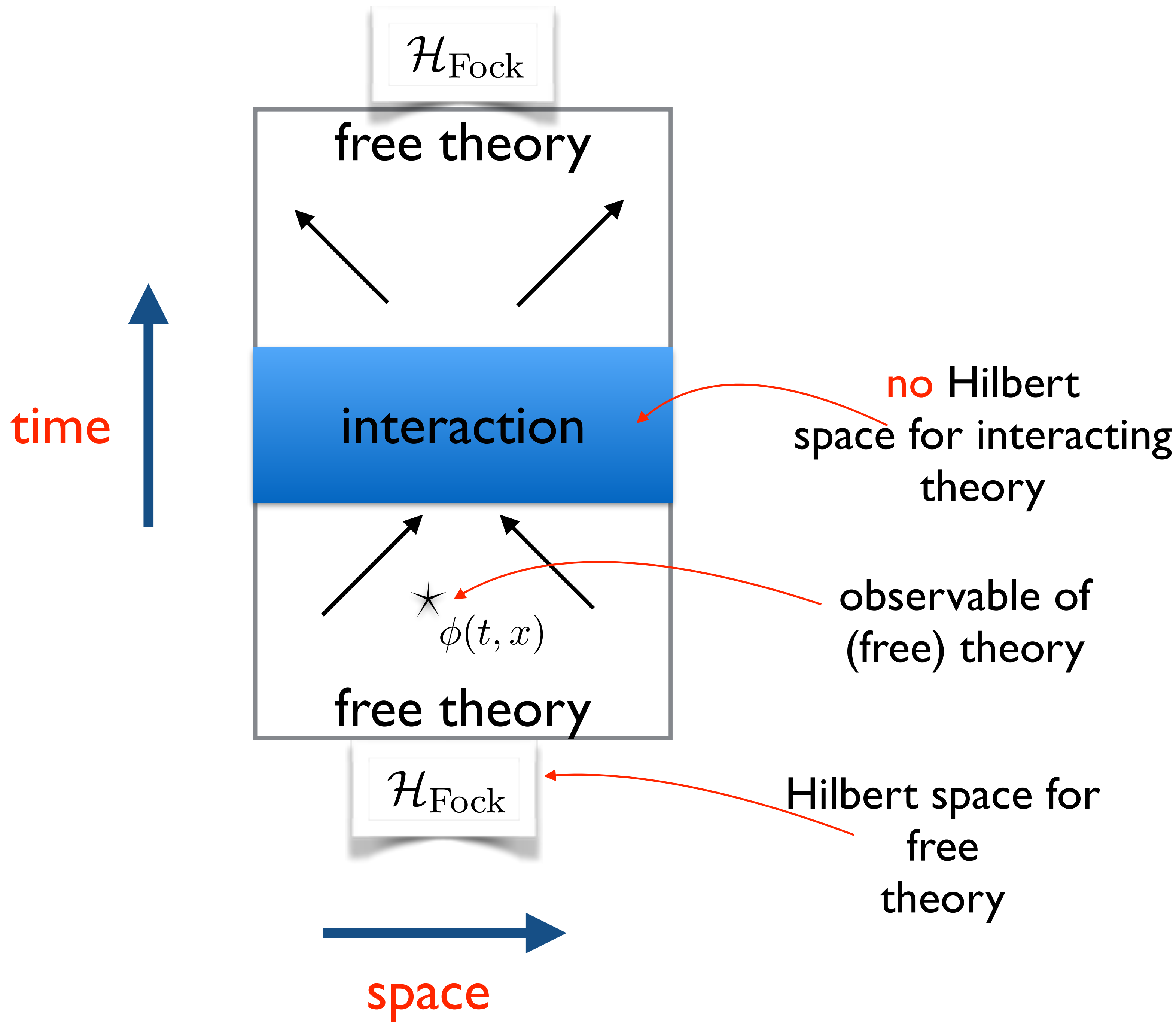
Bianca Dittrich
Perimeter Institute

Problem of Time Workshop
LMU, July 2015

[Milos Luzanin]

What is
Quantum Space Time?

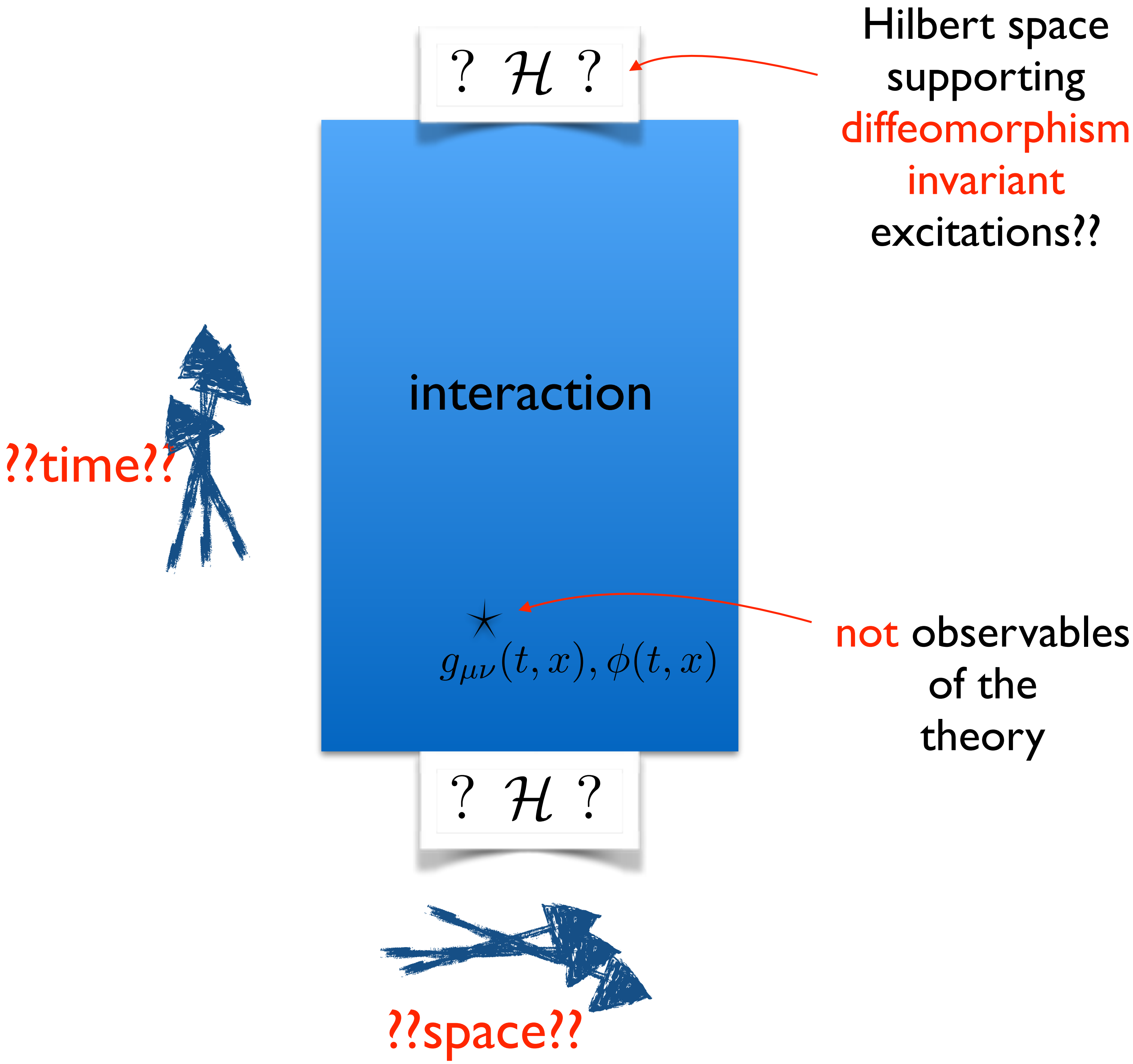
(Perturbative) Quantum Field Theory



Perturbative quantum gravity fails: non-renormalizable.

And does not answer crucial questions (eg big bang).

Quantum gravity



Space time coordinates
have no physical significance.
Need to implement
diffeomorphisms invariance.
This avoids assigning
unphysical
quantum fluctuations to
choice of coordinates.

Quantum geometry dynamics?

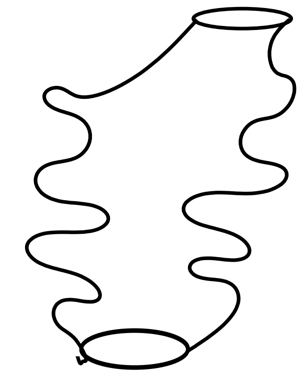
All quantum geometry states encode 4D quantum geometry (histories), however (almost) all of these describe "virtual" (non-dynamical) quantum histories.

Physical states are thus that solve the quantum equations of motion of the theory, aka constraints.

Is it a “problem” with the canonical formalism?

- Start with the path integral approach.

- Want to make geometries quantum: sum over geometries: Geom:
  Need to attach amplitudes to Geom’s. $\mathcal{A}(\mathrm{Geom})$



- Configurations are boundary geometries: b.geom

- Consider wave function(al)s of boundary geometries: $\psi_{\mathrm{out}}(\mathrm{b.geom.out})$ $\psi_{\mathrm{in}}(\mathrm{b.geom.in})$

- Will encode full q-Geom!

Consider “transition amplitude”: give all the observables of the system.

[proof for class. GR: BD 05]

(The interpretation as “transition amplitude” is subtle.)

[Perez,Rovelli]

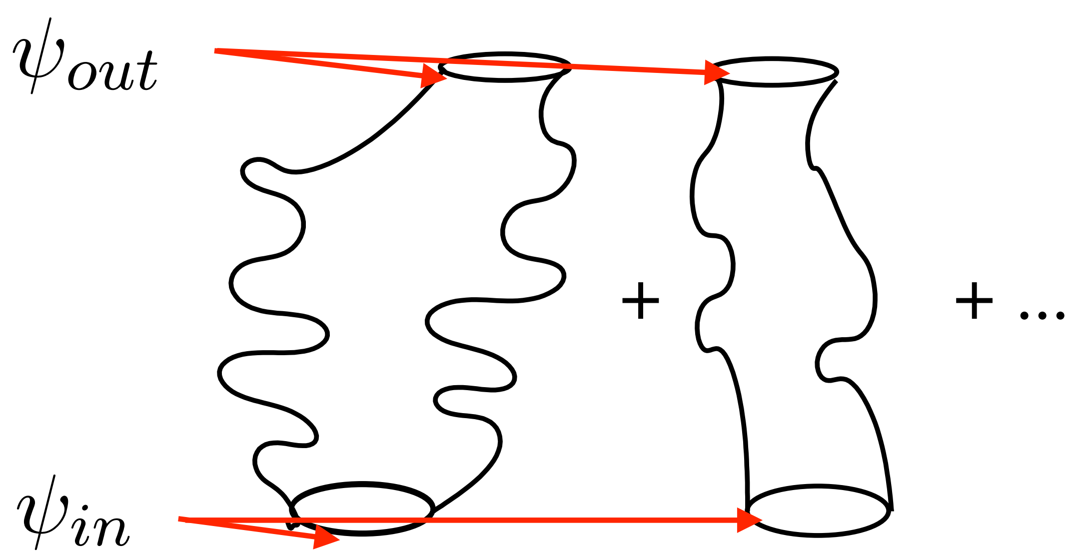
Path integral = sum over spacetime geometries

"transition amplitude"

measure   quantum amplitude

$$\langle \psi_{out} | \mathcal{P} | \psi_{in} \rangle \quad := \quad \int \mathcal{D}(\mathrm{Geom})\ \mathcal{A}(\mathrm{Geom} : (\mathrm{b.geom.in}, \mathrm{b.geom.out}))$$

$$\psi_{\mathrm{in}}(\mathrm{b.geom.in})\ \psi_{\mathrm{in}}(\mathrm{b.geom.out})$$

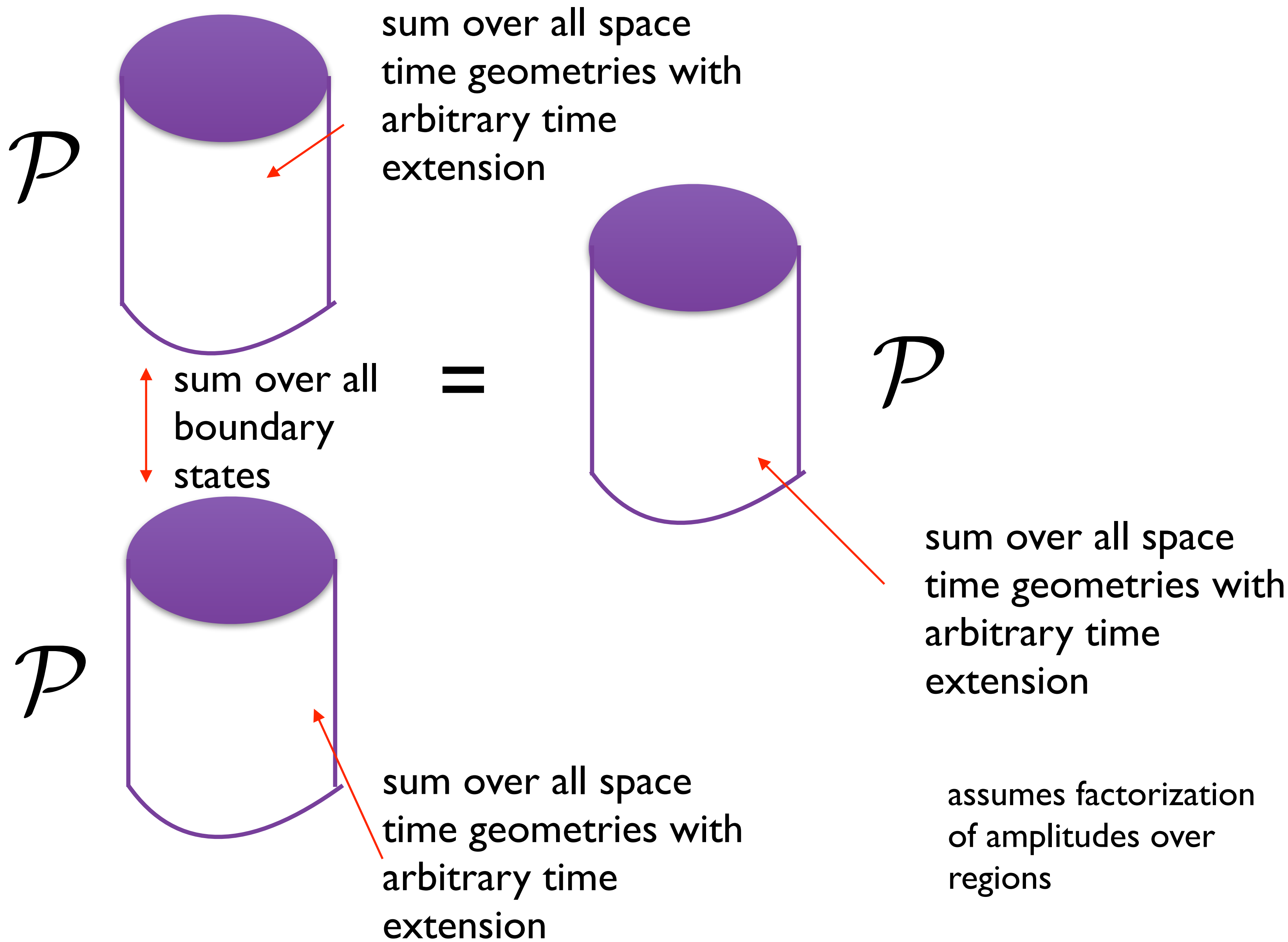


'sum' over geometries

Path integral is a projector

[Halliwell, Hartle 91]

$$\sum_{\text{basis b}} \langle \psi_{\text{out}} \,|\, \mathcal{P} \,|\, \psi_{\text{b}} \rangle \langle \psi_{\text{b}} \,|\, \mathcal{P} \,|\, \psi_{\text{in}} \rangle \;=\; \langle \psi_{\text{out}} \,|\, \mathcal{P} \,|\, \psi_{\text{in}} \rangle$$



assumes factorization of amplitudes over regions

$$\mathcal{P} \circ \mathcal{P} = \mathcal{P}$$

projector property

Path integral is a projector: implies constraints

projector property $\quad \mathcal{P} \circ \mathcal{P} = \mathcal{P}$

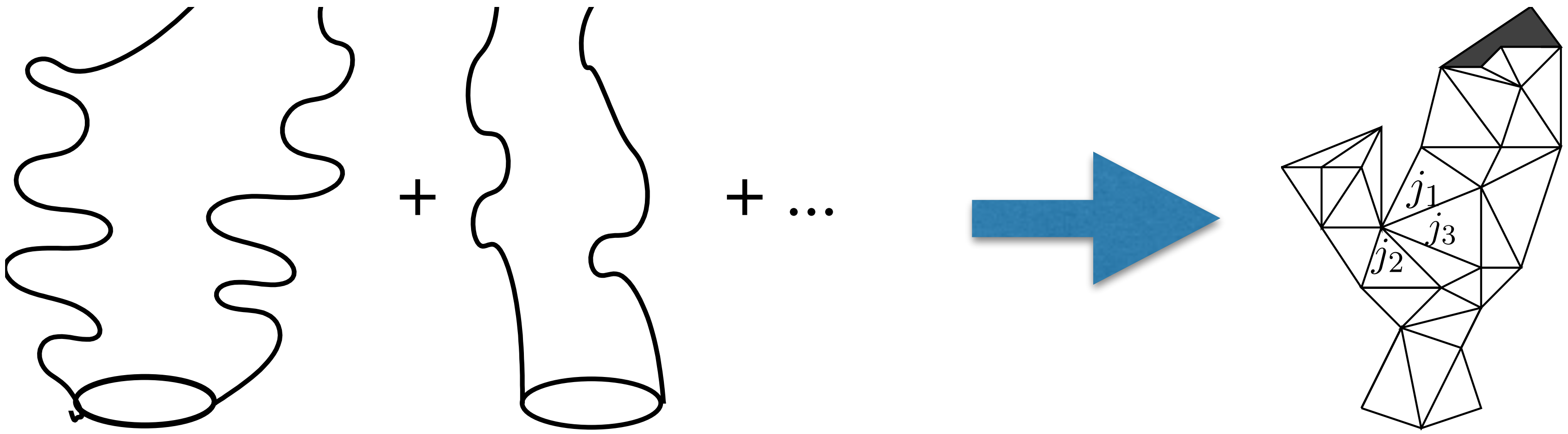
Need only $\quad \psi_{\text{phys}} := \mathcal{P}\psi \quad$ physical states, described by constraints

As $\mathcal{P} = \mathbb{I}$ on physical states there is no time evolution in the usual sense.
Indeed there is no background time parameter in the path integral. "Frozen time picture".

Because we do want to quantize space-time: path integral includes sum over time distances.

Flow of time has to be reconstructed from boundary states.

Path integral and discretization



sum over geometries = sum over labels associated to the triangulation

Path integral not well defined:
What is the path integral measure?

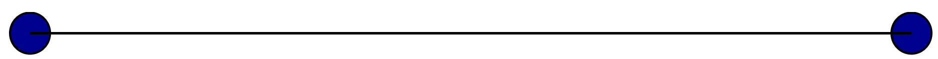
define measure
through discretization of underlying manifold

construction of amplitudes from GR action
→ spin foam model

[Reisenberger, Rovelli, Barrett, Crane, Freidel, Krasnov, Livine, Speziale…]

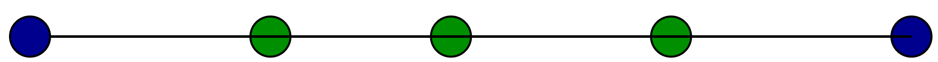
Problem with discretization?

usual path integral for
fixed time interval



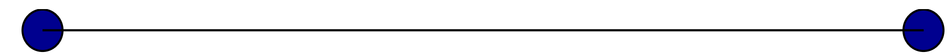
T

discretize and use approximations valid
for small time steps

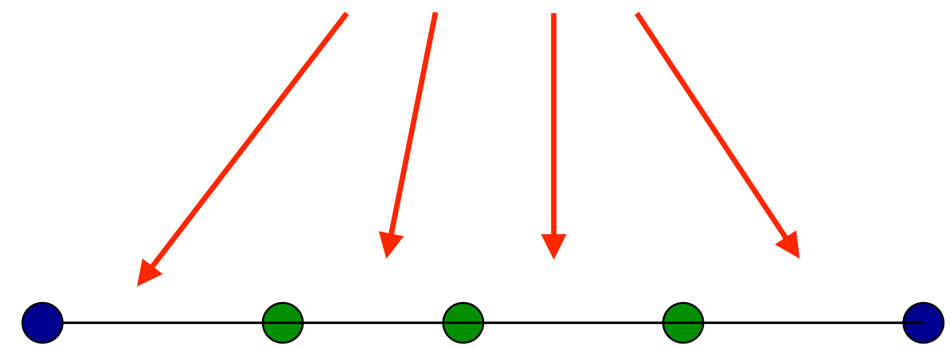


T/N T/N

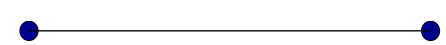
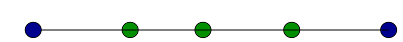
path integral in gravity /
reparam.-invariant systems



can be any time interval, including infinity



In fact

 = 

$$\mathcal{P} = \mathcal{P}^N$$

So how could discretization help??

Problem with discretization?

Discretized path integral in (4D) gravity or reparametrization invariant systems is usually not a projector.

Due to breaking of diffeomorphism symmetry by discretization.

[Bahr, BD 09]
[BD, Hoehn 09]
[Bahr, BD, Steinhaus 11]
[BD, Steinhaus 11]
[BD, Kaminski, Steinhaus 14]

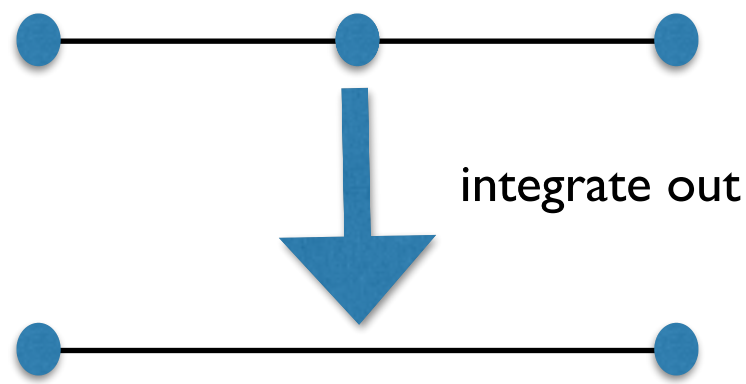
If path integral has projector property: have reached continuum limit (at least in “time”), as further subdivision will not change the result (as path integral is a projector).

Need to restore projector property of path integral!

Evaluating path integral via coarse graining

[Bahr, BD, Steinhaus 11]

$\mathcal{P}_{\text{approx}}$ $\mathcal{P}_{\text{approx}}$



$\mathcal{P}_{\text{better approx}}$

coarse graining flow

$$\mathcal{P} = \mathcal{R}(\mathcal{P}) = \mathcal{P} \circ \mathcal{P}$$

path integral given by fixed point of coarse graining flow:

boundary data encode time interval: take more and more subdivisions into account

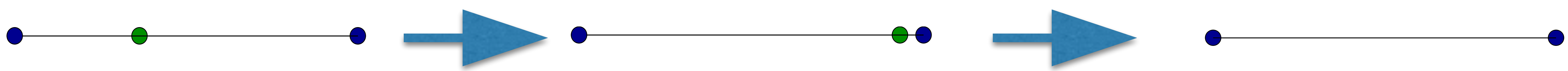
$$\mathcal{P} \simeq \mathcal{P}_{\text{approx}} \mathcal{P}_{\text{approx}} \cdots \mathcal{P}_{\text{approx}}$$

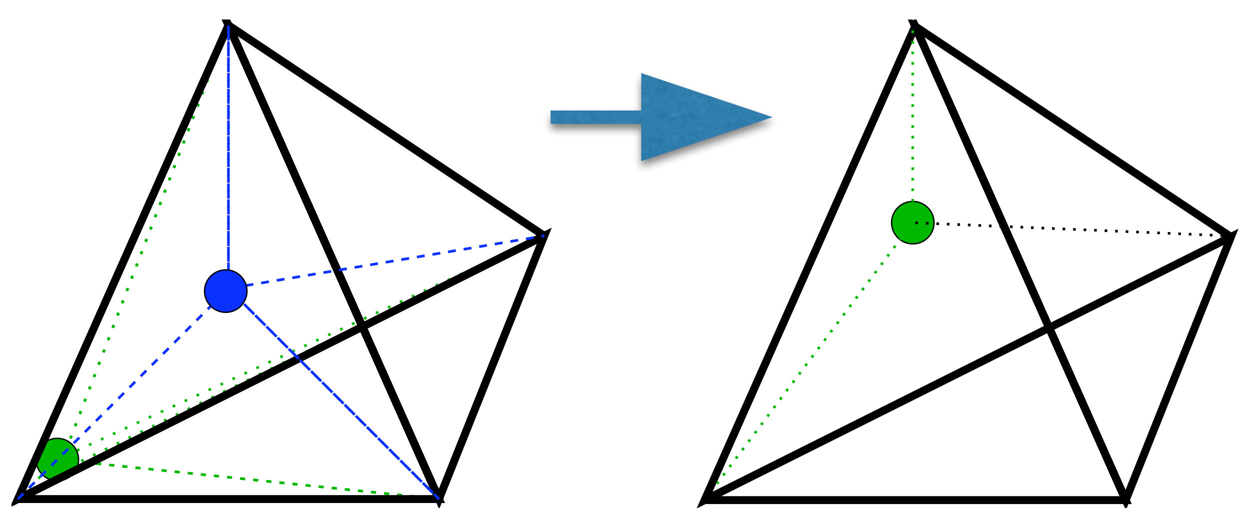
Quality of approximation depends on boundary data (or matrix elements)

Diffeo symmetry and discretization independence restored

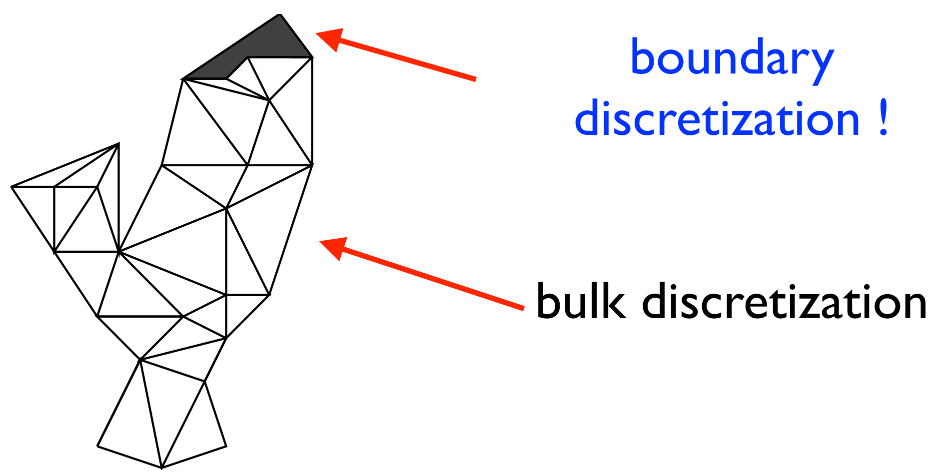
Diffeo-symmetry (in the discrete) implies discretization independence.

[Bahr, BD, Steinhaus 11]





Evaluating path integral via coarse graining: higher dimensions?



Coarse graining of non-topological theories leads to non-local couplings: impossible to control.

[Bahr, BD, He 11]

[BD 12]

Even weak notion of diffeo-symmetry / triangulation independence needs non-local amplitudes.

[BD, Kaminski, Steinhaus 14]

Generalized boundary formalism

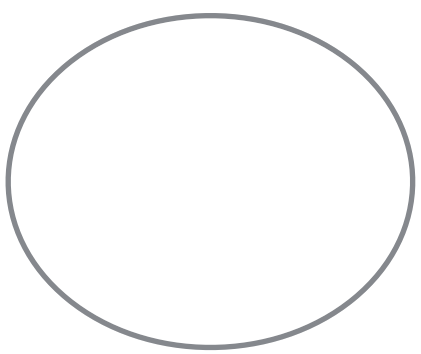
[Oeckl 00's + …]

surface at some constant time

surface at some constant time

$$\mathcal{A}\,(\psi_{\rm in}, \psi_{\rm out}) \quad = \quad \langle \psi_{\rm out} \,|\, \mathcal{P} \,|\, \psi_{\rm in} \rangle$$

Fundamental objects encoding the dynamics.

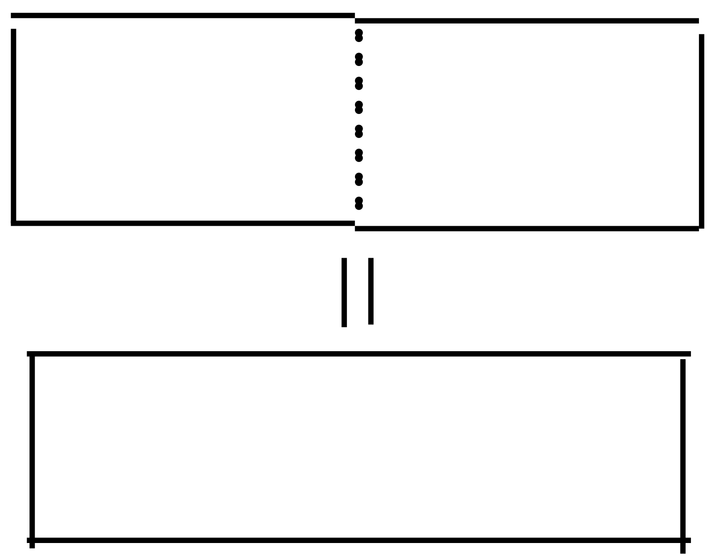


generalized boundary

vacuum=state for an empty boundary
=simplest possible state

$$\mathcal{A}\,(\psi_b) = \langle \psi_b \,|\, \mathcal{P} \,|\, \psi_\emptyset \rangle$$

Generalized boundary formalism

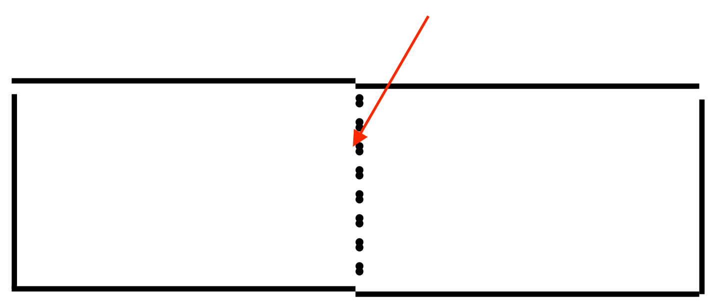


Assumes 'gluing axiom':
needed to get more complicated from simpler amplitudes

~~Amplitude for bigger region~~
~~=~~
~~glued from amplitudes for smaller regions~~

Amplitude for more complicated boundary state
=
glued from amplitudes for less complicated boundary state

Integrating over data associated to discretization misses out on most of the continuum data



However this gluing axiom does not hold in the discrete:
Restricting to discretization we never obtain a full resolution of identity for the continuum Hilbert space.
(This leads eventually to non-local amplitudes if one wants to represent continuum physics.)

Towards consistent boundary formalism

[BD12, BD14, BD to appear]

We do not assume "gluing axiom".

However we need a principle to connect amplitudes for less complicated and more complicated boundary data.

This will also allow to construct the amplitudes in an computation and approximation scheme. [BD, Steinhaus 13]

Main Idea: A "discretization" just determines the wave function for a small subset of the degrees of freedom. All other degrees of freedom are put into the simplest state = vacuum state.

Remarks:

- One can choose what the vacuum is.
- In this formalism we are discrete and continuous at once: discreteness just means to probe finitely many degrees of freedom.
- However we still have to 'emulate' continuum dynamics.

Main Challenge: Be consistent - observables should not depend on choice of discrete structure, which is used to compute it.

To be discrete or not discrete …

[Ashtekar-Lewandowski-Isham representation of loop quantum gravity]

[New representation! BD, Geiller 14a,14b, Bahr, BD, Geiller 15]

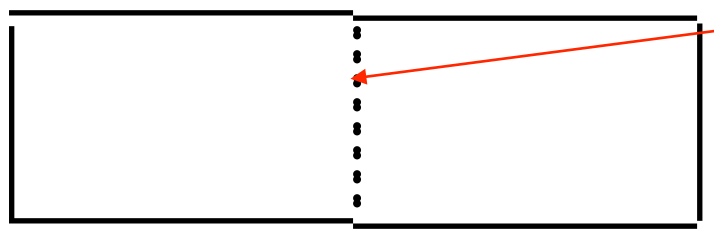
nice summer read!

Inductive limit Hilbert space

See discrete structure as a probe of continuum Hilbert space.

Simple states can be represented on simple (discrete) structures. More complicated states require more complicated discretization.

Cylindrical consistency conditions ensure that observables do not depend on choice of discrete structure.

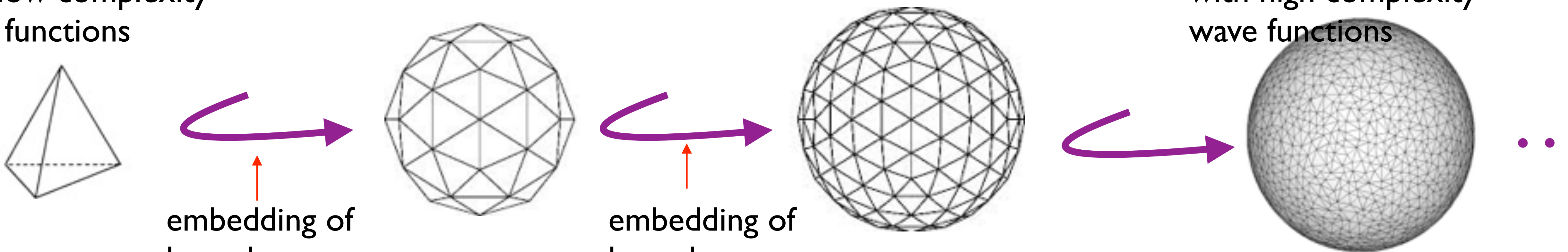


cannot obtain
full resolution of unity in
inductive limit Hilbert space
if we do not consider all
states (on arbitrary fine
discretizations)

How to express the continuum dynamics [BD NJP 12, 14]

Boundary Hilbert space
with low complexity
wave functions

Boundary Hilbert space
with high complexity
wave functions



embedding of
boundary
Hilbert spaces

embedding of
boundary
Hilbert spaces

restricts to

$$\mathcal{A}_{vac}^{\text{low com}}(\psi_{\text{low com}}) \longleftarrow \mathcal{A}_{vac}^{\text{med com}}(\psi_{\text{med com}}) \longleftarrow \mathcal{A}_{vac}^{\text{high com}}(\psi_{\text{high com}}) \cdots$$

(cylindrical) consistency condition

A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

[BD NJP 12, 14]

A new paradigm: consistent boundary formalism

[BD12, BD14, BD to appear]

Instead of gluing axiom, which cannot hold in the discrete,
we require
consistency of the amplitudes as functionals on the (inductive limit) Hilbert space.

Can this be used to compute the amplitudes?

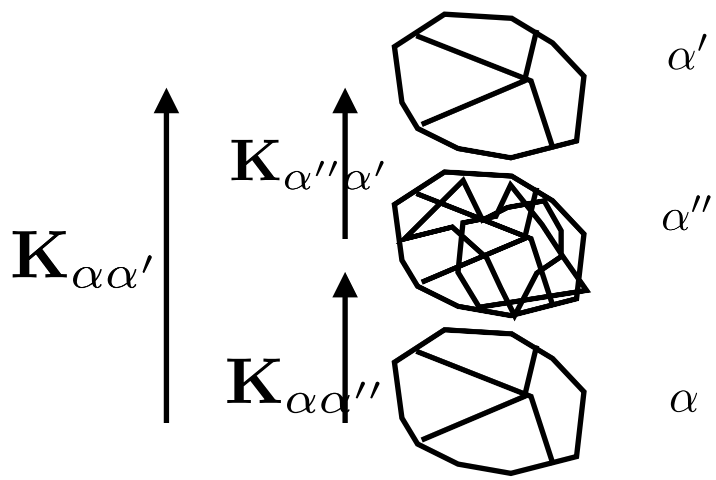
Yes. [BD12, BD14, BD to appear]

Amplitudes can be computed iteratively in an approximation scheme.

Least effort necessary for low complexity = homogeneous 'cosmology' configurations.

Constructing amplitudes

- Construct amplitude for simplest boundary (i.e. simplex) as a first approximation to final answer.
- Us this amplitude in the usual gluing scheme to build amplitudes for more complicated 'transitions'.



$$\mathbf{K}_{\alpha''\alpha'} \circ \mathbf{K}_{\alpha\alpha''} = \mathbf{K}_{\alpha\alpha'}$$

convolution property

$$\mathcal{A}_{\alpha'}(\iota_{\alpha\alpha'}\psi_\alpha) \quad = \quad \langle\psi_\emptyset|(\mathbf{K}_{\emptyset\alpha'})^\dagger|\mathbf{K}_{\alpha\alpha'}\psi_\alpha\rangle \quad = \quad \mathcal{A}^{imp}_{\alpha}(\psi_\alpha)$$

(dual) vacuum amplitude

coarse graining (by time evolution) of this amplitude

Iterative process: coarse graining

$$\mathcal{A}_{\alpha'}(\iota_{\alpha\alpha'}\psi_\alpha) \quad = \quad \langle \psi_\emptyset | (\mathbf{K}_{\emptyset\alpha'})^\dagger | \mathbf{K}_{\alpha\alpha'}\psi_\alpha \rangle \quad = \quad \mathcal{A}^{imp}_\alpha(\psi_\alpha)$$

(dual) vacuum amplitude

coarse graining (by time evolution) of this amplitude

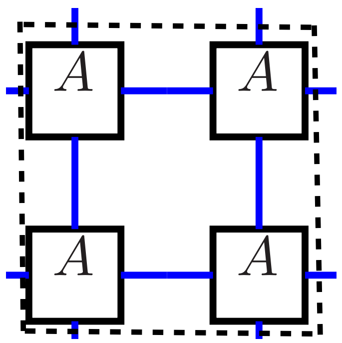
Fixed point of iteration process satisfies consistency by construction:

$$\begin{aligned} \mathcal{A}_{\alpha'}(\iota_{\alpha\alpha'}\psi_\alpha) \quad &= \quad \langle \psi_\emptyset | (\mathbf{K}_{\emptyset\alpha'})^\dagger | \mathbf{K}_{\alpha\alpha'}\psi_\alpha \rangle \\ &\overset{\text{Conv}}{=} \quad \langle \psi_\emptyset | (\mathbf{K}_{\emptyset\alpha})^\dagger | \psi_\alpha \rangle \quad = \mathcal{A}_\alpha(\psi_\alpha) \quad . \end{aligned}$$

In praxis: tensor network renormalization

(using local truncation method)

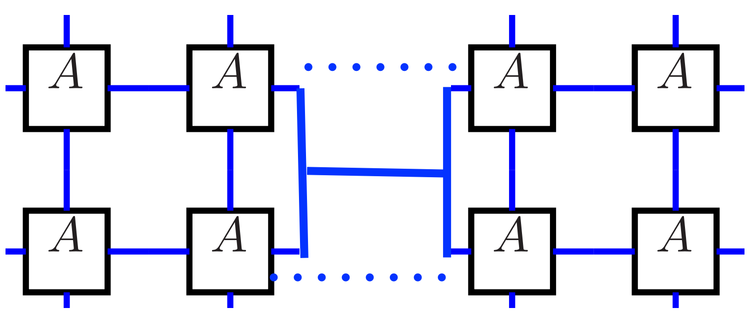
Coarse grain



bare/initial amplitude
depending on four variables

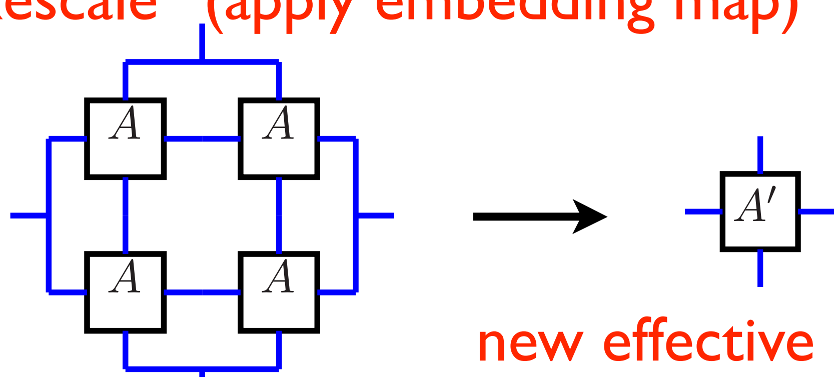
Contract initial amplitudes (sum over bulk variables). Obtain "effective amplitude" with more boundary variables.

Truncate /determine embedding map



Find an approximation (embedding map) that would minimize the error as compared to full summation (dotted lines). For instance using singular value decomposition, keeping only the largest ones.
Leads to field redefinition, and ordering of fields into more and less relevant.

"Rescale" (apply embedding map)

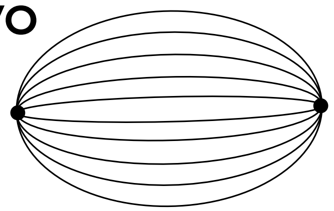


new effective amplitude

Use embedding maps to define coarse grained amplitude with the same (as initial) number of boundary variables.

Phase diagram for spin foam analogues

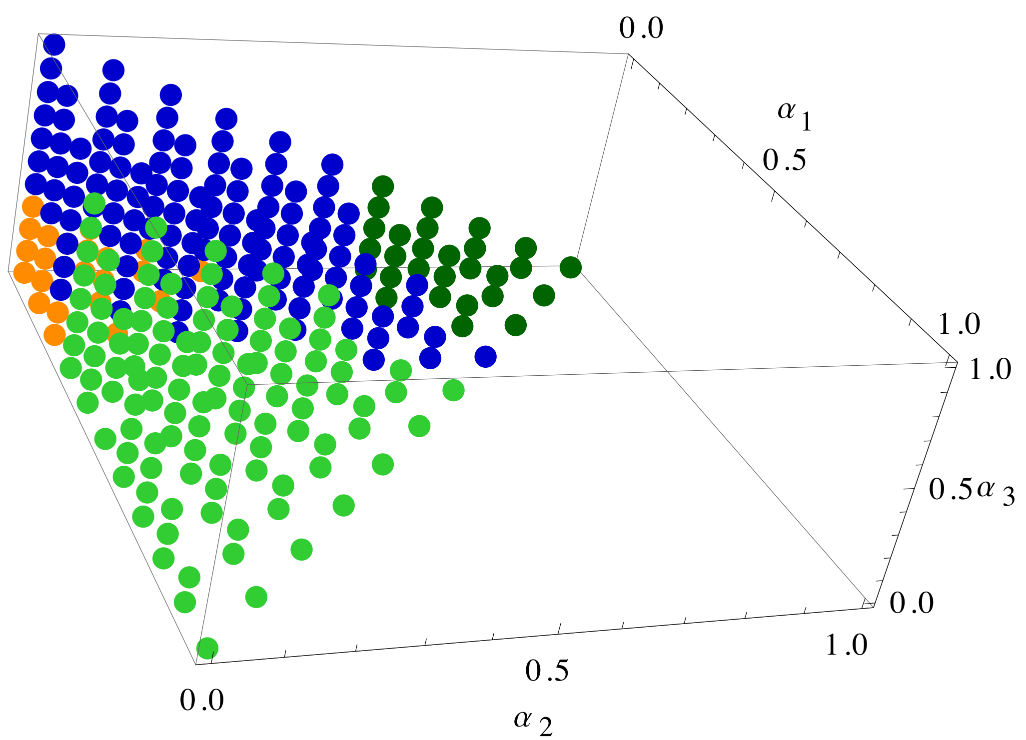
- models are similar to anyonic spin chains [Feiguin et al 06]
- but can be also interpreted as particular spin foams describing the gluing of two space time atoms



- changing certain parameters in initial model: changes how the atoms glue

  (technically: changes implication of simplicity constraints)

- anyonic spin chains support very rich phase structure, classification in [BD, Kaminski 13 and to appear]



Interpretation: different phases describe uncoupled space time atoms (green) and coupled space time atoms (orange,blue).

Positive indication for finding a geometric phase in spin foams.

[BD, Martin-Benito, Schnetter NJP 13]

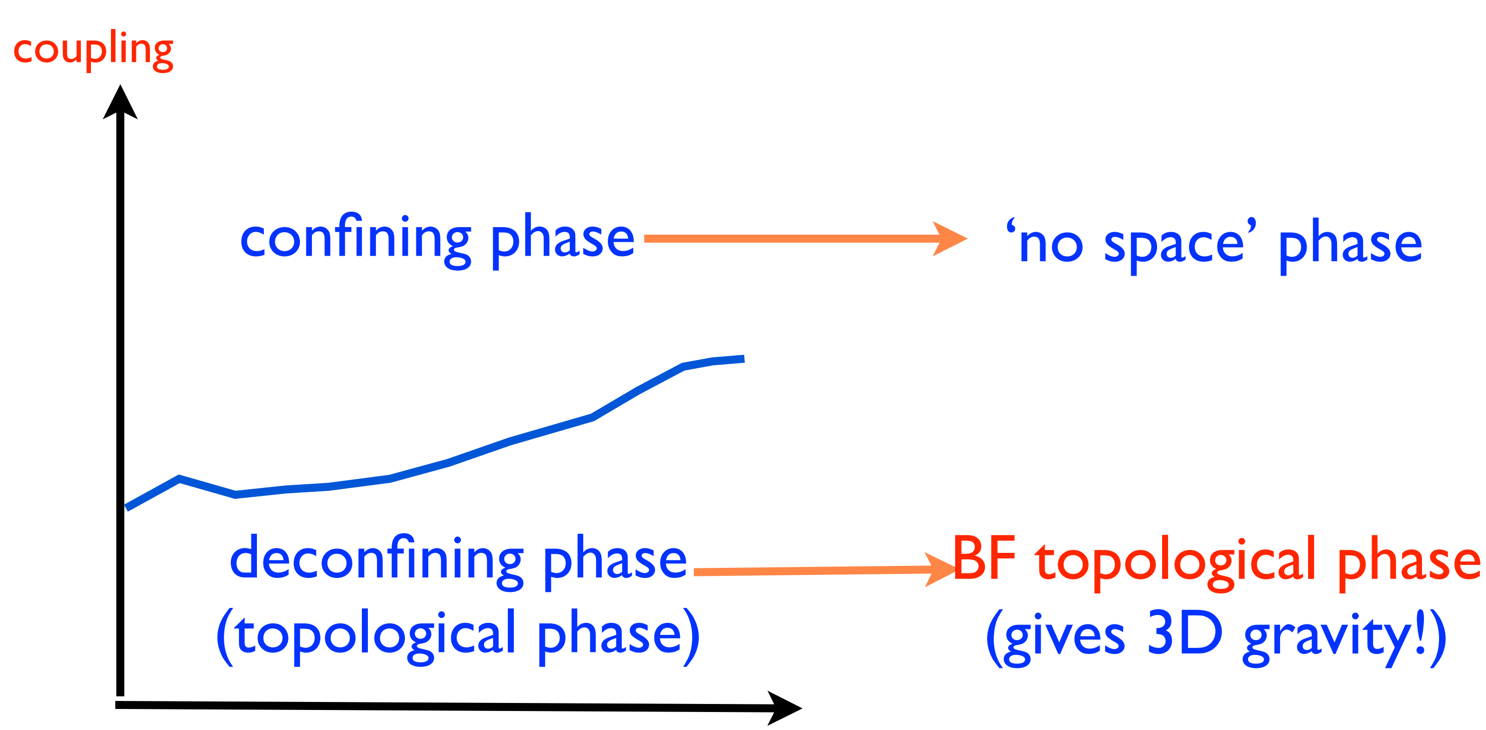
BD, Martin-Benito, Steinhaus PRD 13]

Phase diagram for spin foams ?

- need to develop (tensor network) coarse graining algorithms for

  spin foams = generalized lattice gauge theories
- first algorithm for 3D Abelian lattice gauge theories: decorated tensor networks
  [BD, Mizera, Steinhaus 14]
- 3D Non-Abelian lattice gauge theories [Delcamp, BD to appear]

Phases in lattice gauge theory

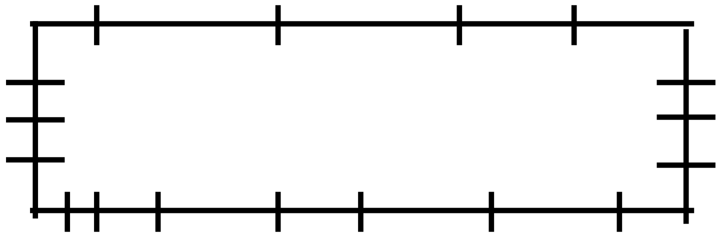


Are there more phases in spin foams?

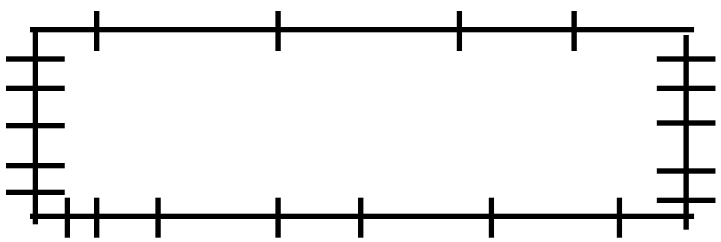
Positive indication from

2D analogue models.

Complexity of states and flow of time



things might happen



more things might
happen

physical vacuum:
nothing happens
(with respect to homogeneous state
described by vacuum)

There is a lot to say and explore but Philipp rather wants me to finish and lead you to a chaotic universe. Even quantum.

Where there would be even more to say about …

Summary

- Quantum space-time:
  - time evolution operator is a projector
  - interpretation reconstructable from boundary data

- Consistent boundary formalism:
  - new paradigm to express full (continuum) dynamics and probe it with (lower complexity) boundary states
  - allows for systematic calculation and approximation scheme

- wip: Construction of Quantum Space Times