In Favour of a Schrödinger Evolution for the Universe

Sean Gryb with Karim Thébault

Radboud Universiteit Nijmegen
Institute for Mathematics, Astrophysics
and Particle Physics

The Problem of Time in Perspective
Munich Center for Mathematical Philosophy
July 3, 2015

Fundamental Postulate

It is possible, in principle, to measure time using a clock.

$\Rightarrow$ clock readings are observables.

Negative Argument for Reduced Phase Space and Dirac Quantization

Clock readings are observable

$\Downarrow$

Global reparametrizations do not reflect a redundancy at the level of phase space variables.

Space of Solutions

What the space of solutions is:

- The space of valid initial data for the classical evolution problem.
  $\Rightarrow$ meaningless without a notion of evolution.

AND NOT

- A physical (i.e., reduced) phase space (no evolution).
  $\Rightarrow$ consequence of a redundancy.

Thus:
A quantization of the space of solutions does not capture the physics of our Fundamental Postulate $\Rightarrow$ clocks not treated as observable.
This applies to:

- Reduced phase space quantization.
- Dirac quantization.

Our Proposal (In Brief)

For globally reparametrization invariant theories:

Relational Quantization

- Observables (mutables): Full non-redundant phase space, (q, p).
- Classical evolution: integral curves of the Hamiltonian constraint, $\mathcal{H}$, labelled by unobservable parameter τ

$$\dot{q} = \{q, \mathcal{H}\} \qquad \dot{p} = \{p, \mathcal{H}\}$$

- Quantum evolution: mutable algebra evolves according to

$$\hat{q}(\tau) = e^{-i\tau\hat{\mathcal{H}}}\hat{q}(0)e^{i\tau\hat{\mathcal{H}}} \qquad \hat{p}(\tau) = e^{-i\tau\hat{\mathcal{H}}}\hat{p}(0)e^{i\tau\hat{\mathcal{H}}}$$

$\Rightarrow$ Time Dependent Schrödinger equation.

For gravity:

Shape Dynamics

- Fix local time by requiring local scale invariance $\Rightarrow$ Shape Dynamics.
- Relationally quantize Shape Dynamics.

The "Global" in Global Reparametrization

Invariance of Action

Action $S = \int \mathrm{d}\tau L(q, \dot{q})$ invariant under

$$\tau \to f(\tau)$$

Note:

- τ - Single time variable (not a function of Σ).
- q - can be a field on Σ.

Caveat

L - homogeneous of degree 1 in $\dot{q}$.
$\Rightarrow$ Hamiltonian constraint $\mathcal{H} = 0$.

Classical solutions are integral curves of $\mathcal{H}$ labelled by τ.

Reduced Phase Space Quantization

Note: will distinguish from deparametrization.

Observation

Classical histories = integral curves of $\mathcal{H}$

$$\Downarrow$$

- Quotienting flow of $\mathcal{H}$ $\Rightarrow$ space of initial data.
- No Hamiltonian flow on reduced phase space.

$\Rightarrow$ cannot generate classical solutions. (Clocks are not redundancies.)
$\therefore$ inappropriate for quantization. (Only appropriate for a genuine redundancy.)

Space of initial data is isomorphic to, but not have the same representational capacity as, the space of histories.

Deparametrization Definition

Note: will consider distinct from phase space reduction.

Nomenclature

Partition phase space into:

- potential clocks (partial observables).
- Dirac observables of a particular clock (complete observables).

Procedure: (Donald's talk)

- Choose a clock $T \equiv q_i \in q$
- Identify $T = \tau$ (special gauge fixing of $\mathcal{H} = 0$)
- Find Dirac observables $Q_I = q_I$, $P_I = p_I$ for $(I \neq i)$.
- The (Q, P) are the complete observables with respect to the partial observable T.
- Solve $\mathcal{H} = 0$ for $E \equiv p_i = H_i$.
- Use H_I to evolve (Q, P).

$\Rightarrow$ Gives (correct) classical evolution of (Q, P) in terms of T.

Deparametrization Difficulties

Leads to effective evolution.

But...

- Good clocks are hard (/impossible) to find. (E.g., Philipp's talk)
- Dirac observables don't tell the whole story.
  $\Rightarrow$ T is not a Dirac observable. (Spacetime scalars??)
- T has an ambiguous interpretation (Rovelli: observable but not predictable).
- Wrong physical picture for Yang–Mills (phase space reduction is).

Difficulties are interpretational in classical theory, but physical in quantum.

Quantum Deparametrization Definition

Choose a particular deparametrization wrt T:

- Complete (Dirac) observables:

$$(Q, P); \{Q, P\} = \mathbb{1} \qquad \Rightarrow \qquad (\hat{Q}, \hat{P}); [\hat{Q}, \hat{P}] = i\hbar\hat{\mathbb{1}}$$

- Evolution:

$$E = H_T(Q, P, T) \qquad \Rightarrow \qquad -i\hbar\frac{\partial\psi}{\partial T} = \hat{H}_T(\hat{Q}, \hat{P}, T)$$

$\Rightarrow$ gives evolution of $(\hat{Q}, \hat{P})$ in terms of clock parameter T.

Quantum Deparametrization Difficulties

Leads to effective notion of evolution.

But...

- Good clocks are hard (/impossible) to find.
  $\Rightarrow$ Serious (/insurmountable) ordering ambiguities.
- Different clock choices are unitarily inequivalent.
  $\Rightarrow$ T takes classical values, but is an operator for different clock choice.

- Conflict: Dirac observables violate our Fundamental Principle.
- The status of T is physically restrictive in quantum theory.

$\Rightarrow$ Need a way to represent all phase space functions as quantum observables AND recover the correct classical limit.

Classical Ontology

Proposal

- Identify all non-redundant phase space functions (q, p) with classical observables $\equiv$ *mutables*.
- Evolution of mutables generated by $\mathcal{H}$:

$$\dot{q} = \{q, \mathcal{H}\} \qquad \dot{p} = \{p, \mathcal{H}\}$$

- Solutions are curves on phase space parametrized by unobservable label τ.
- Can use solution to express evolution of any mutable in terms of any other.

$\Rightarrow$ ontological shift: clock variables are considered observable.

Advantages

- Phase space evolution is integrable in τ.
- Can handle "winding numbers".
- Role of clock is non-ambiguous.

Quantum Proposal

- Mutables become operators that define the quantum observable algebra.

$$(q,p);\{q,p\}=\mathbb{1} \qquad \Rightarrow \qquad (\hat{q},\hat{p});[\hat{q},\hat{p}]$$

- Evolution of mutables generated by $\hat{\mathcal{H}}$:

$$\hat{q}(\tau)=e^{-i\hbar\tau\hat{\mathcal{H}}}\hat{q}(0)e^{i\hbar\tau\hat{\mathcal{H}}} \qquad \hat{p}(\tau)=e^{-i\hbar\tau\hat{\mathcal{H}}}\hat{p}(0)e^{i\hbar\tau\hat{\mathcal{H}}}$$

  τ - arbitrary unobservable label.

- Equivalent to Schrödinger evolution

$$-i\hbar\frac{\partial\psi}{\partial\tau}=\hat{\mathcal{H}}(\hat{q},\hat{p})\psi$$

Note: $\hat{\mathcal{H}}(\hat{q},\hat{p})$ is τ-independent
$\therefore$ "energy" (or some coupling) promoted from coupling constant to conserved quantity.
$\Rightarrow$ classical limit reproduced (energy eigenstate is superselected in classical limit)

Advantages

- Evolution is unitary in τ.
- All potential clocks can be described quantum mechanically.
- Unifying framework for partial/complete observables program:
  $\Rightarrow$ If $\hat{q}_i = \hat{T}$ is such that $\hat{T} \approx \langle \hat{T} \rangle \hat{\mathbb{1}}$ then we reproduce standard clock deparametrization
- Our Fundamental Principle is implemented.

Shape Dynamics

- Split time diffeomorphisms into:
  1. Hypersurface preserving (i.e., global reparametrizations)
  2. Hypersurface deforming (i.e., local refoliations)
- Identify:
  1. Global reparametrization $\Rightarrow$ evolution
  2. local refoliations $\Rightarrow$ redundancy

Observation

$\exists$ global time such that non-redundant (reduced) phase space is locally scale invariant.

Ontological shift: scale invariance suggests preferred foliation.
$\Rightarrow$ gravity expressed in terms of the dynamics of scale invariant local "shapes":

$$\boxed{\text{One damn shape after another!}}$$

Relational Quantization of Gravity

- Use Shape Dynamics to fix global time.
  $\Rightarrow$ non-redundant phase space is fixed by imposing local scale invariance.
- Relationally quantize Shape Dynamics.

Summary/Outlook

Summary

- Fundamental Principle (FP): Time can, in principle, be measured by a clock.
  $\Rightarrow$ Clock values are observable.
- Standard deparametrization in conflict with FP.
  $\Rightarrow$ clocks are not Dirac observables.
- Promote clocks to observables (mutables).
- Mutables evolve unitarily wrt unobservable parameter.
- Can reproduce classical limit, implement FP, and not privilege any clock choice quantum mechanically.

Outlook

- Mini-superspace quantization.
- Nabu–Goto string quantization: Feynman versus Hadamard propagator.

Thank You!