First-Class Constraints, Gauge, and the Wheeler–DeWitt Equation

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Words of Wisdom?

That the (canonical) quantization of general relativity leads to a timeless formalism should be understood as a consequence of an incorrect treatment of the temporal symmetries of the classical theory. By treating local temporal labellings as entirely unphysical, and change as entirely relational, we do not retain in the quantum formalism the full classical dynamics or the implicit temporal-ordering structure.

(Gryb and Thébault, 2015, 5)

Plan

1. Orthodoxy
2. Pitts's Challenge
3. Barbour and Foster's Challenge
4. Morals

Plan

1. ~~Orthodoxy~~ Widespread Belief
2. Pitts's Challenge
3. Barbour and Foster's Challenge
4. Morals

What is Orthodoxy?

- First-class constraints generate gauge transformations
- "as generating functions of infinitesimal contact transformations, [the primary first-class constraints] lead to changes in the *q*'s and the *p*'s that do not affect the physical state" (Dirac, 1964, 21)
- "one postulates that different phase space points x_1, x_2 describe the same physical state if they are connected by a gauge transformation. Here a gauge transformation is a transformation which is generated by the constraints…" (Dittrich, 2007, 1894)

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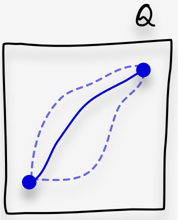
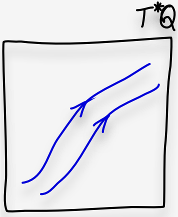
The view is particularly clear from a geometrical perspective...

The Geometrical Perspective on Hamiltonian Mechanics





$T^*\mathcal{Q}$ is a symplectic manifold (M, ω). We can use ω to associate a vector field X_f with any given function f on M via:

$$\omega(X_f, \cdot) = df.$$

We can then define the Poisson Bracket:

$$\{f, g\} := \omega(X_f, X_g) = \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p^i} - \frac{\partial f}{\partial p^i}\frac{\partial g}{\partial q_i} \right)$$

Phase space curves represent physically possible histories iff they are the integral curves of X_H, where $\omega(X_H, \cdot) = dH$.

The evolution of an arbitrary quantity, f, is given by:

$$\dot{f} = \{f, H\}.$$

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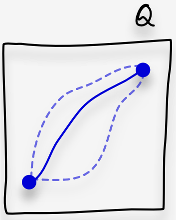

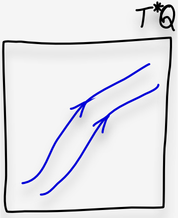

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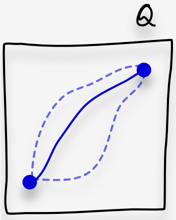

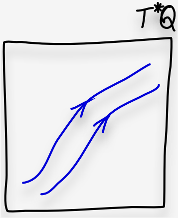

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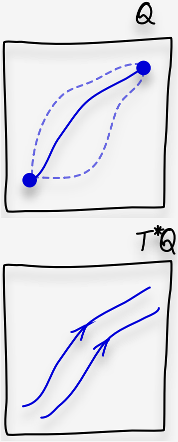

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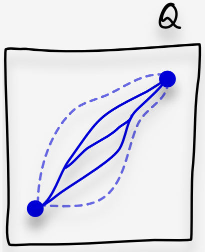
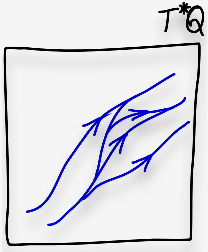
Constrained Hamiltonian Dynamics

If a theory's Lagrangian is (quasi-)invariant under the action of a group parametrized by arbitrary functions of the independent variables:

- the p^i are not independent but must satisfy "primary" constraints $\varphi_n(p,q) = 0$, so
- The Hamiltonian dynamics lives on a proper subspace of $T^*\mathcal{Q}$, the "constraint surface", defined by $\varphi_n(p,q) = 0$ (and any "secondary" constraints).





Orthodoxy (Geometrical Version)

- Restricting the canonical symplectic form ω on $T^*\mathcal{Q}$ to the constraint surface N defines a presymplectic form σ on N.
- x and y lie in the same gauge orbit iff they are connected by a curve whose tangent vector is everywhere in the kernel of σ.
- One can view as dynamical trajectories integral curves of X_H where $\sigma(X_H, \cdot) = \mathrm{d}H$ and H is an appropriate a gauge-invariant function on N.
- The dynamics is indeterministic, but the dynamics of gauge-invariant quantities is deterministic: only functions on N that are constant on gauge orbits correspond to genuine physical magnitudes.

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What is the connection between this geometrical perspective and the dynamics framed in terms of the Poisson bracket on the full phase space $T^*\mathcal{Q}$?
(Lagrangian equivalence *vs* the "extended" Hamiltonian)

Different Notions of Gauge

Gauge$_1$ x and y are points of a presymplectic manifold that lie in the same gauge orbit (as defined previously).

Gauge$_2$ x and y represent the same physical state/history

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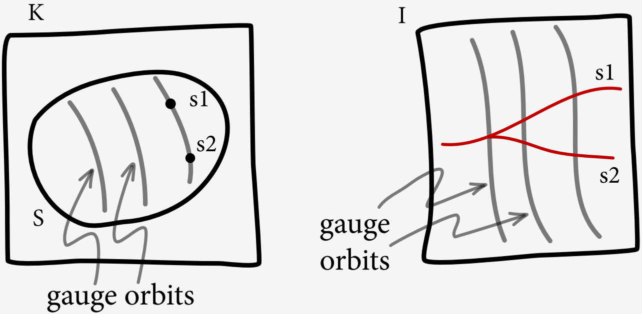

- one treats gauge$_1$-related points of $\mathcal{I}$ as gauge$_2$-related, *therefore*
- one treats gauge$_1$-related solutions, s_1 and s_2, as gauge$_2$-related.

Some Examples

1. A "Leibnizian" particle theory
(≈ 1977 Barbour–Bertotti theory with absolute time):
   - $L = V - T_{\mathrm{BB}}$, where

$$V = -\sum_{i<j} \frac{m_i m_j}{r_{ij}} \quad \text{and} \quad T_{\mathrm{BB}} = \sum_{i<j} \frac{m_i m_j}{r_{ij}} \left(\frac{\mathrm{d}r_{ij}}{\mathrm{d}t} \right)^2$$

$$r_{ij} := |\mathbf{q}_i - \mathbf{q}_j|$$

   - Arbitrary time-dependent rotations and translations as gauge symmetries.
   - Points within a gauge orbit of $\mathcal{I}$ correspond to the same relative particle configuration.
2. Electromagnetism (?)

PoT: Applying Orthodoxy to GR is Prima Facie Absurd

- The Hamiltonian of GR is a sum of first-class constraints (it is a “totally constrained” Hamiltonian system): all points on any given dynamical trajectory lie within the same gauge orbit.
- A function that commutes with all the first class constraints will commute with the Hamiltonian: no gauge-invariant quantity takes on different values at different points of a dynamical trajectory.

The task: a clear understanding of why orthodoxy is not applicable to GR.

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Challenge 1

Pitts, J. B. (2014). "A first class constraint generates not a gauge transformation, but a bad physical change: The case of electromagnetism." *Annals of Physics* **351**, 384–406.

Hamiltonian Electromagnetism

- $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- $H_c = \int \mathrm{d}\mathbf{x}\,[\frac{1}{2}(\vec{\pi}^2 + \vec{B}^2) + \vec{\pi}\cdot\nabla A_0]$

π^μ are the variables canonically conjugate to A_μ. Defined in terms of the Lagrangian, $\pi^\mu = -F^{0\mu}$, so:

- $\vec{\pi}$ is the electric field, and
- $\pi^0 \approx 0$ is a primary constraint.

Stability of this constraint under the Hamiltonian dynamics leads to a secondary constraint:

- $\dot{\pi}^0 = \{\pi^0, H_c\} = \nabla\cdot\vec{\pi} \approx 0.$

Both constraints are first class.

The π^μ are left unchanged by transformations generated by the constraints.

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Pitts (2014): FCCs Generate Bad Physical Changes

Transformations generated by π^0

$$\delta A_\mu(x) = \left\{ A_\mu(x), \int d^3y \pi^0 \xi(t,y) \right\} = \delta^0_\mu \xi(t,x), \text{ so}$$

$$\delta F_{\mu\nu} = \partial_\mu \xi \delta^0_\nu - \partial_\nu \xi \delta^0_\mu, \text{ and so}$$

$$\delta F_{0n} = -\delta \vec{E} = -\partial_n \xi.$$

In general $\nabla \cdot \vec{E} = 0 \mapsto \nabla \cdot \vec{E} + \nabla^2 \xi \neq 0$.

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One obtains $\delta F_{0n} = 0$ by setting $\xi = -\dot{\varepsilon}$.
This is the form taken in electromagnetism of the "gauge generator", discussed in the work of Castellani, and Pons, Shepley and Salisbury. One has, for $G = \int d^3x(\pi^i_{,i}\varepsilon - \pi^0\dot{\varepsilon})$:

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Does this undermine orthodoxy?

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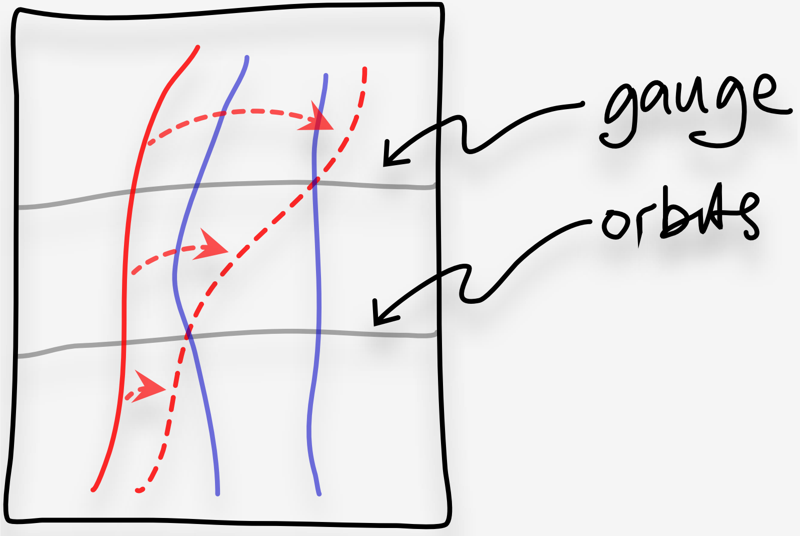
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orbits

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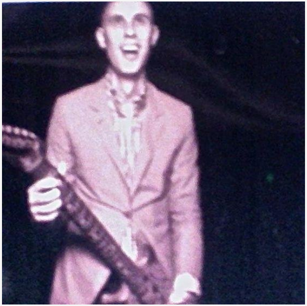

Barbour, Julian, and Brendan Z. Foster. "Constraints and gauge transformations: Dirac's theorem is not always valid." arXiv preprint arXiv:0808.1223 (2008).

Dirac's Argument

Consider the infinitesimal change in some quantity g after a short time δt.

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g(\delta t) &= g_0 + \dot{g}\delta t \\
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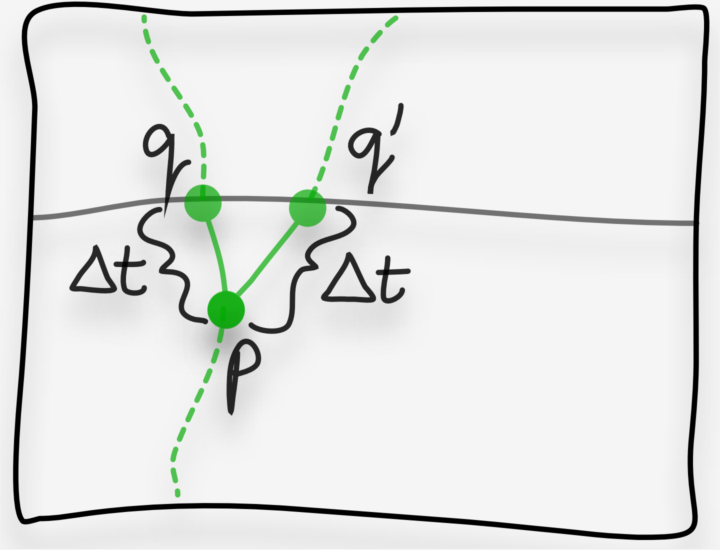
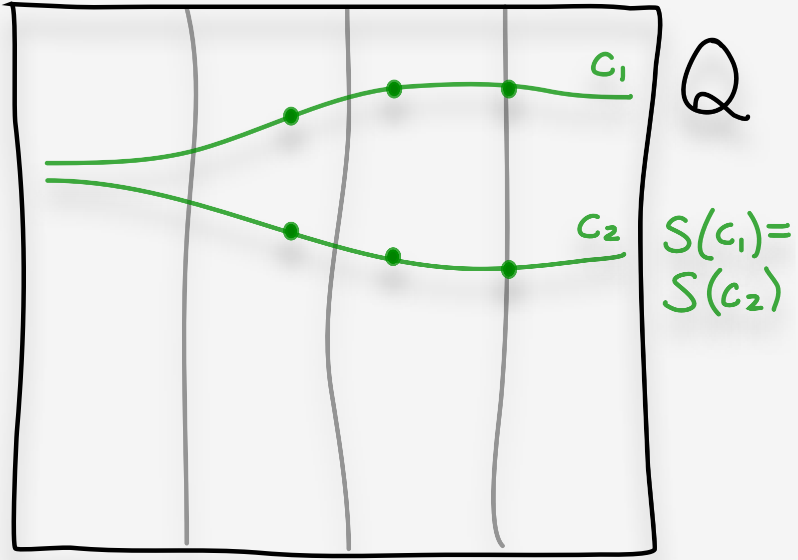


Varieties of Gauge Redundancy

Configuration space redundancy



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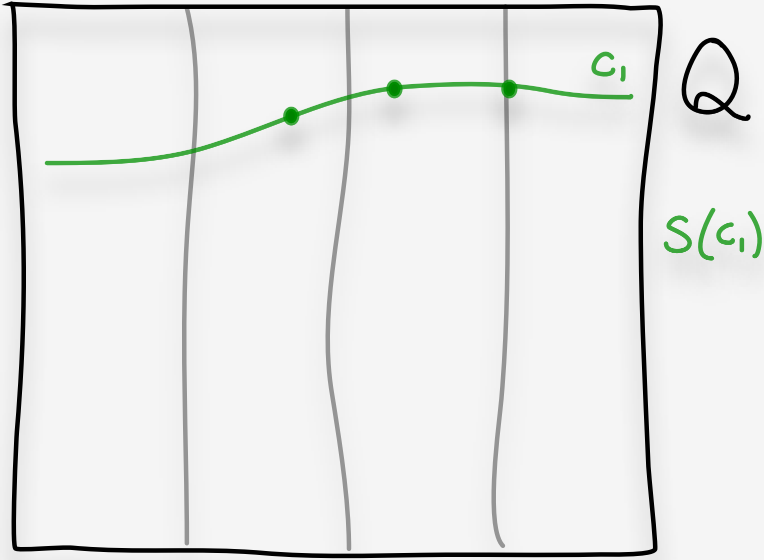

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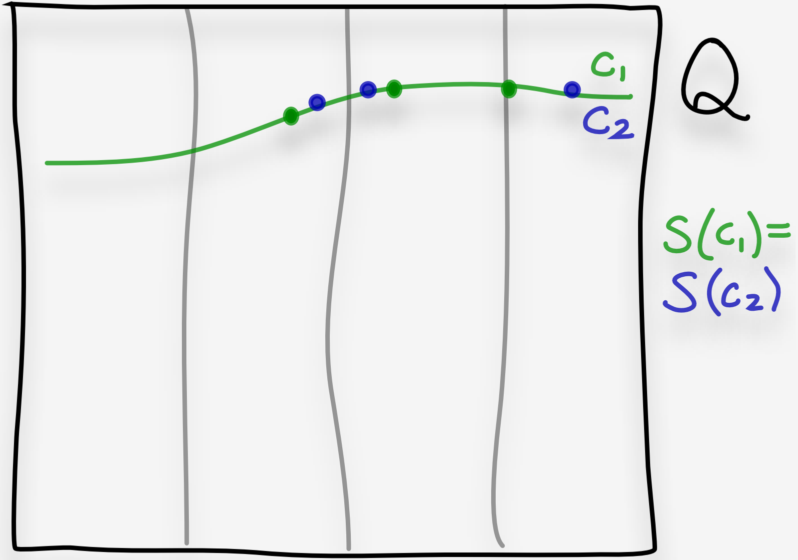

Moral: one need not identify "gauge-related" points

Finally, a word of caution. The arguments leading to the identification of [the first class constraints] as generators of transformations that do not change the physical state at a given time implicitly assume that the time t…is observable. That is information brought in from the outside. One may also take the point of view that some of the gauge arbitrariness indicates that the time itself is not observable. This is done in so-called generally covariant theories…One of the arbitrary functions is then associated with reparametrizations $t \longrightarrow f(t)$ of the time variable.

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A Simple Reparamatrization Invariant Theory

Jacobi's Principle:

$$I_J = \int_A^B L_J = 2\int_A^B d\lambda \sqrt{(E-V)T}$$

The Hamiltonian is given by:

$$H = \sum_i \mathbf{p}_i \cdot \dot{\mathbf{q}}_i - L_J = Nh, \text{ where}$$

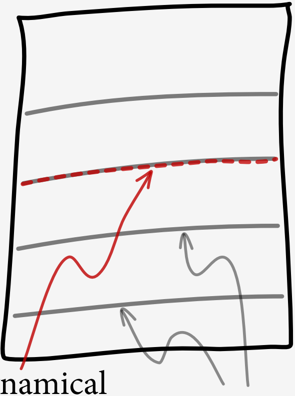
$$N = \sqrt{\frac{T}{E-V}} \quad \text{and} \quad h = \frac{1}{2}\sum_i \mathbf{p}_i \cdot \mathbf{p}_i + V - E$$

Given the definition of $\mathbf{p}_i$,

$$\mathbf{p}_i = \dot{\mathbf{q}}_i / N \qquad h \approx 0$$

h is a primary first-class constraint.

No Indeterminism at the Level of Phase Space



- Dynamical trajectories are integral curves of X_h, where $\sigma(X_h, \cdot) = \mathrm{d}h$, but $h \approx 0$.
- X_h does two things:
  - defines a *path* in $\mathcal{I}$
  - provides a parameterization.
- Distinct X_h define the same paths.
- There is no apparent indeterminism at the level of paths
  - No gauge redundancy in functions *on phase space*.
- Only the evolution of such quantities as functions of *parameter time* is undetermined.

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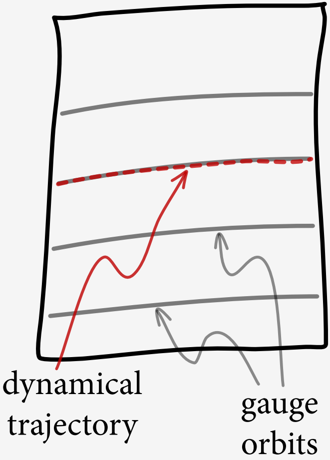

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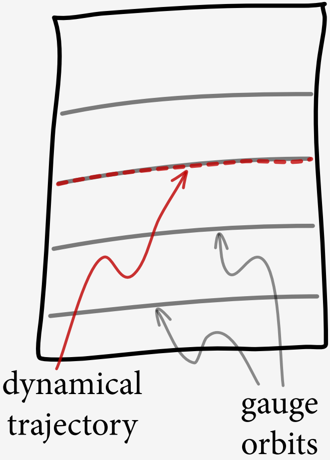
No Indeterminism at the Level of Phase Space



- Dynamical trajectories are integral curves of X_h, where $\sigma(X_h, \cdot) = dh$, but $h \approx 0$.
- X_h does two things:
  - defines a *path* in $\mathcal{I}$
  - provides a parameterization.
- Distinct X_h define the same paths.
- There is no apparent indeterminism at the level of paths
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Types of Gauge Transformation: Recap

- Maps from points to physically equivalent points
- Maps that leave images of solution curves invariant but change the paramaterization

There is no pressure (from the requirement of determinism) to require that genuine physical magnitudes weakly commute with the constraint that generates the latter.

Unfortunately there can be a third type of gauge redundancy: maps that map paths to distinct, but physically equivalent, paths without mapping points to physically distinct points.

The problem of refoliation invariance.

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Aside: this is also the right way to think about theories *without* gauge redundancy (when interpreted as cosmological theories).

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Earman on Coincidence Quantities

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No, that's exactly how the measuring procedure works.

Against the Proposal

The problem is that all of our observations must be expressed in terms of the physically measurable quantities of the theory, namely those combinations of the dynamic variables which are [gauge invariant and therefore] independent of time. One cannot try to phrase the problem by saying that one measures the gauge dependent variables, and then looks for time independent correlations between them, since the gauge dependent variables are not measurable quantities within the context of the theory. (Unruh, 1991, 266)

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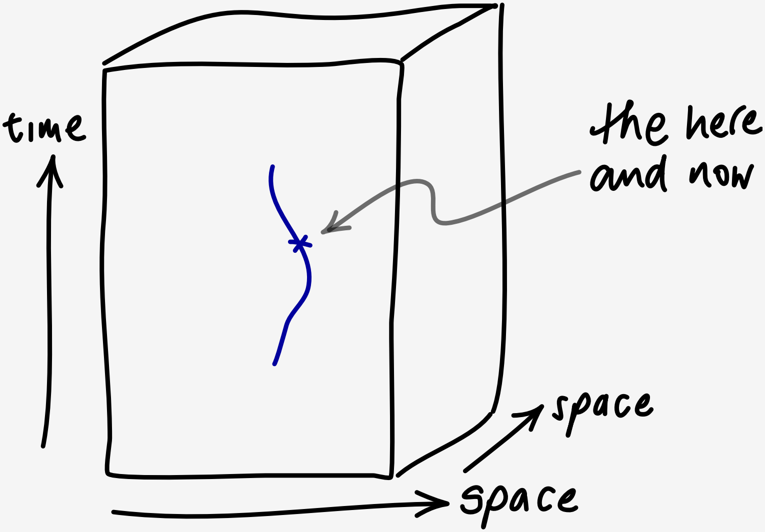
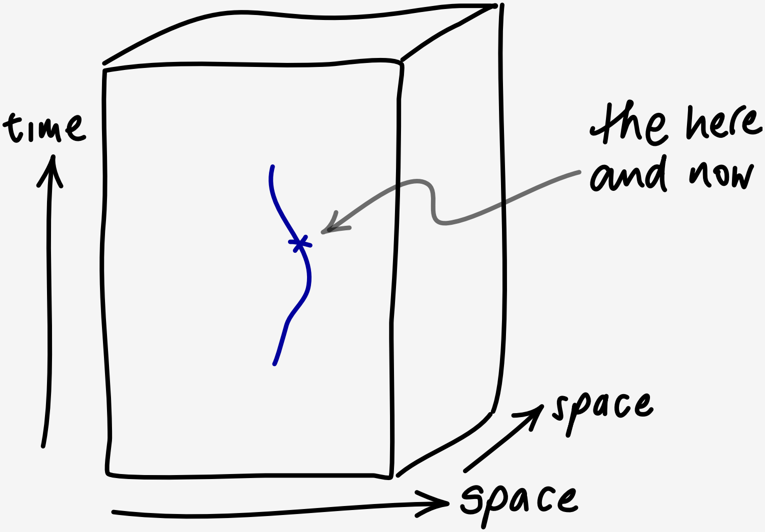
Plan

1. Orthodoxy
2. Pitts's Challenge
3. Barbour and Foster's Challenge
4. Morals

The Block Universe





This is not a case of “emergent time”!

The Wheeler–DeWitt Equation

- Apply Dirac constraint quantization to Hamiltonian GR
- Physical states are those invariant under transformations generated by the quantum constraints.
- The result…

$$H|\Psi\rangle = 0$$

- Which notion of gauge does this presuppose?

What's wrong with a naively temporal (Everettian) understanding of transition probabilities between components of $|\Psi\rangle$?

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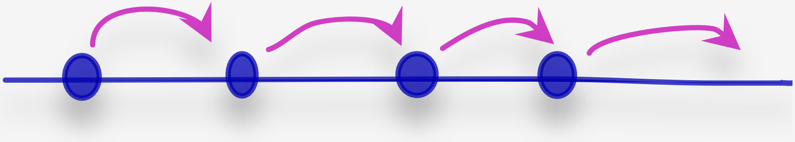

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