

Physics without fundamental time

Carlo Rovelli

- I. Time, who?
- II. Physics without fundamental time
- III. A real world example

What are we talking about
when we talk about
time?

Aristotle :

(Changing) Substance

Descartes:

Res extensa

Newton:

Time

Space

Particles

Faraday-Maxwell:

Fields

Particles

Einstein 05:

Spacetime

Einstein 15:

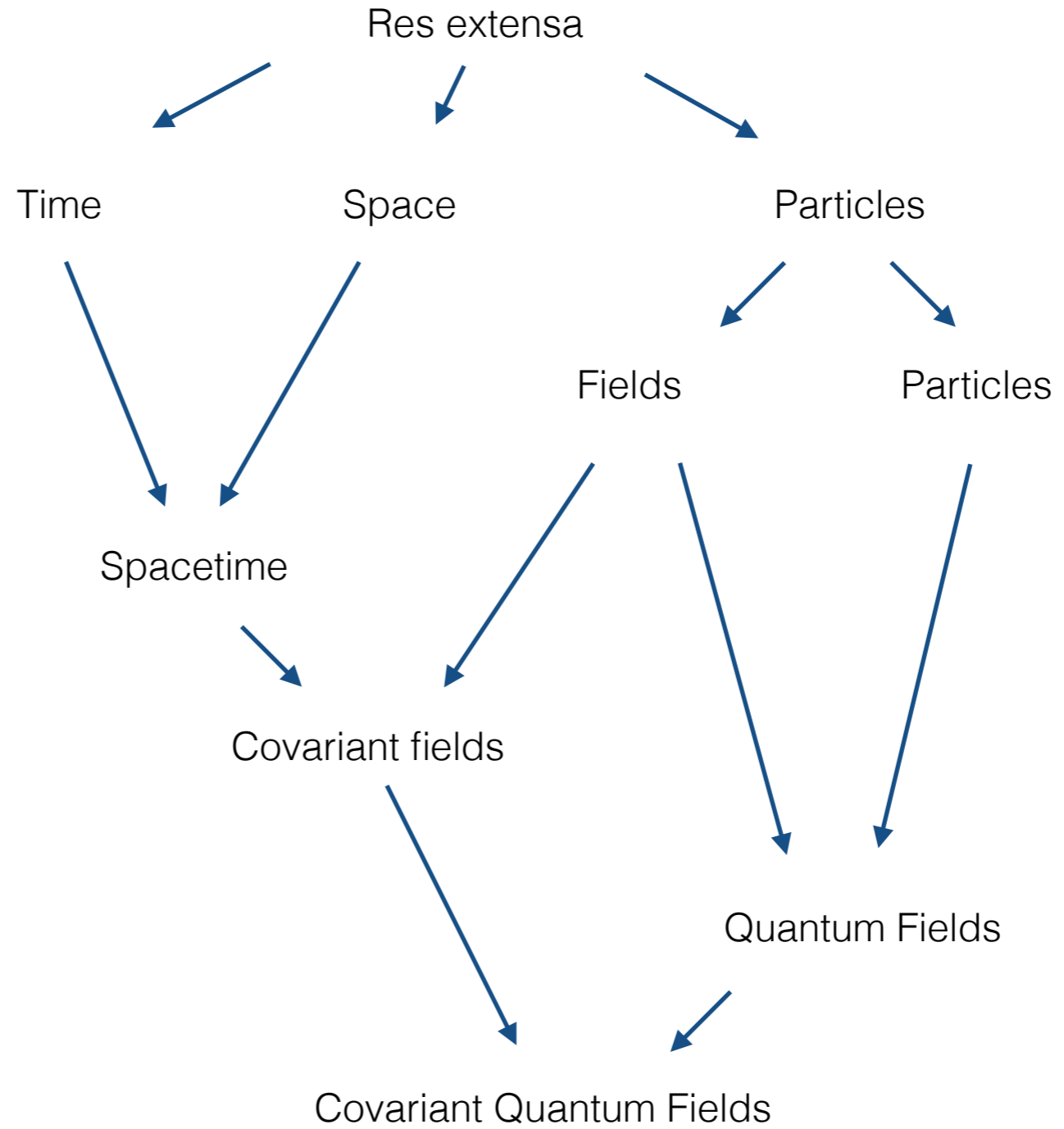
Covariant fields

Quantum:

Quantum Fields

Quantum gravity?:

Covariant Quantum Fields

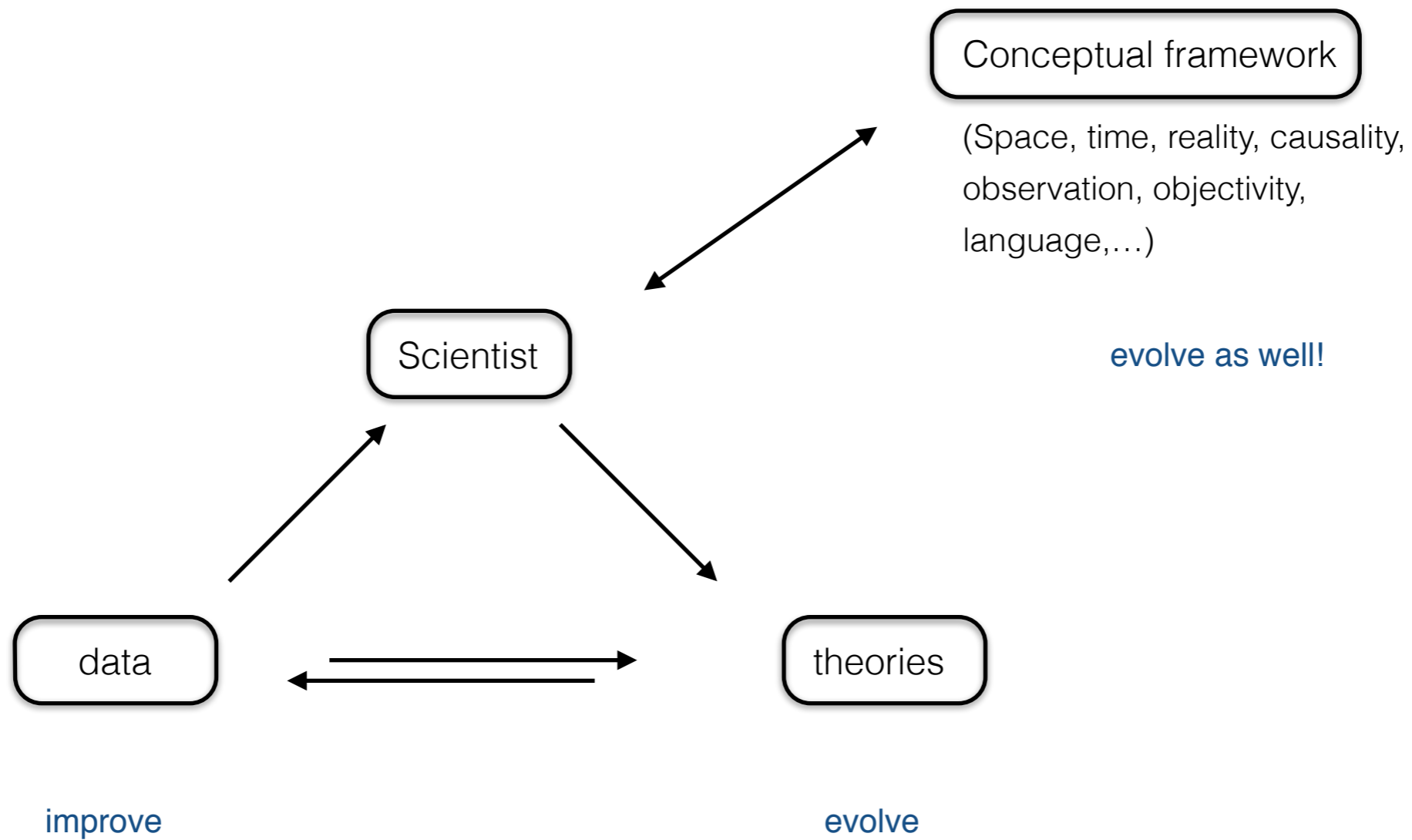


Change is “the actuality of that which potentially is, qua such”

(Aristotle Physics 201a9-11)

Time is “a number of change”

(Aristotle Physics 219b1)



Non foundationalism: - There is no *final* scientific theory of the world.
- There is no *definitive* metaphysics.

We learn.

“On the basis of general relativity, space, as opposed to “what fills space”,
has no separate existence.
If we imagine the gravitational field to be removed, there does not remain a space [...],
but absolutely nothing.

A Einstein, *Relativity, the special and general theory*, 1917

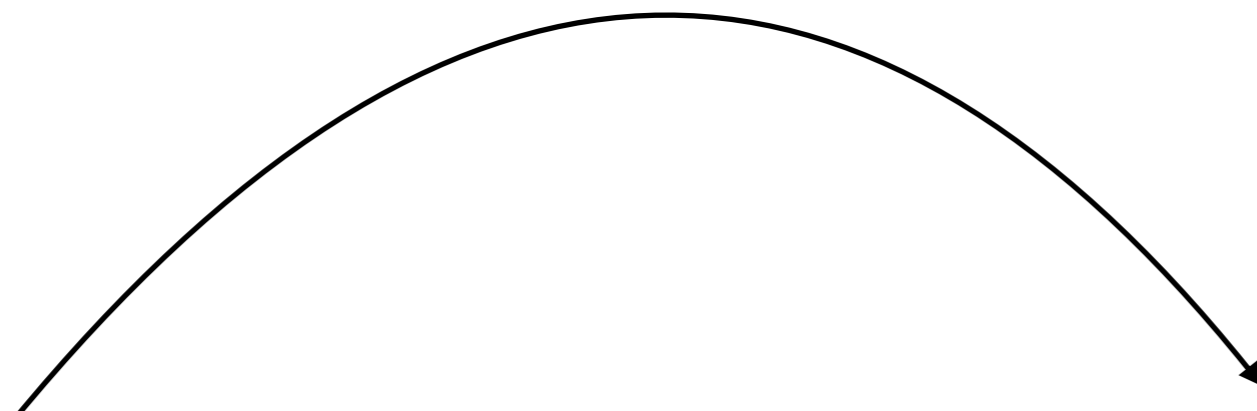
General relativity “takes away from space and time
the last remnant of physical objectivity”

A Einstein, *The foundations of general relativity*, 1916

$$\text{Time} = \int_{\gamma} d\tau \sqrt{g_{\mu\nu}(\gamma(\tau)) \frac{d\gamma^{\mu}}{d\tau} \frac{d\gamma^{\nu}}{d\tau}}$$



gravitational field



T₂



T₁

$$\Delta T = T_2 - T_1$$

Times in physics:



Thermodynamical (and common sense):

oriented, metric, universal

Classical and quantum mechanical:

metric, universal

Special relativistic, quantum field theoretical:

metric, frame dependent

Dynamics on curved spacetime:

metric, path dependent

Dynamics of gravitational field:

relational

Quantum gravity:

none



Special initial conditions
or special observer



Low relative velocities



Low curvature



Negligible backreaction

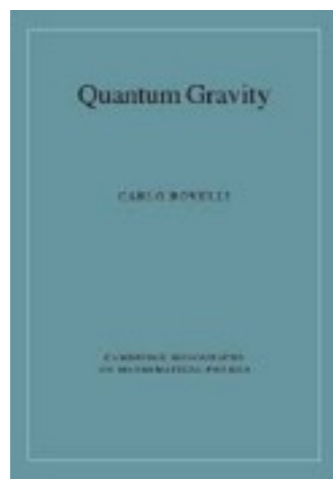


Negligible quantum gravity effects

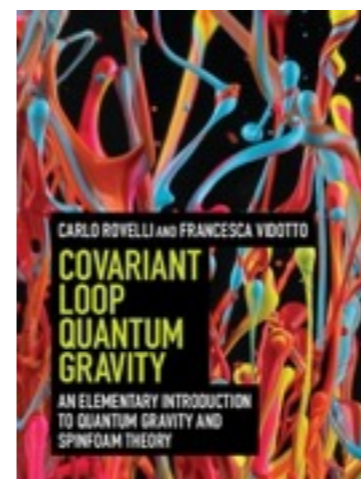
Can we do physics,
without time?

Of course yes!

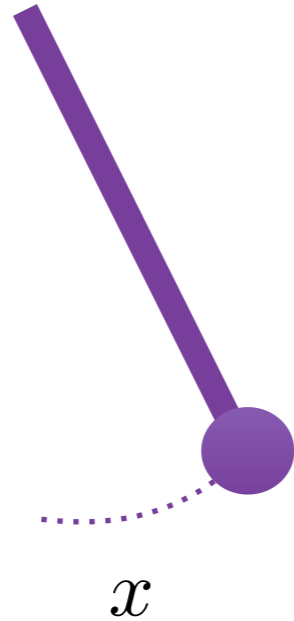
Lagrange, ...,
Dirac, Souriau, Arnold,...
DeWitt,
...



Quantum gravity
Carlo Rovelli
CUP 2004



**Introduction to Canonical
Loop Quantum Gravity**
Francesca Vidotto, Carlo Rovelli
CUP 2014



t

Position (dependent variable):

$$x \in \mathcal{C}$$

Time (independent variable):

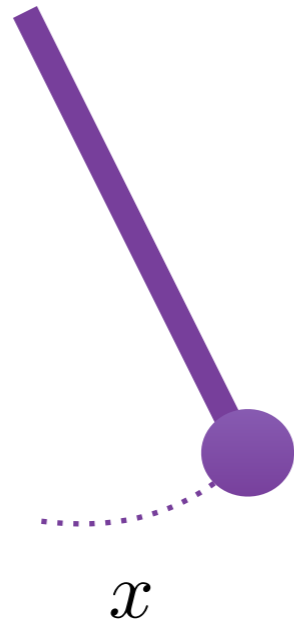
$$t \in \mathcal{R}$$

Equations of motion:

$$\frac{d^2 x(t)}{dt^2} = -\omega^2 x(t)$$

General solution (motions):

$$x(t) = A \sin(\omega t + \phi)$$



partial observables

Cfr: Sean
Cfr: Brian

Variables (dependent and independent not distinguished):

$$(x, t) \in \mathcal{E} = \mathcal{C} \times \mathcal{R}$$

Equations of motion:

$$dx - p dt = 0; \quad dp + \omega^2 x dt = 0$$

General solution (motions):

$$f(x, t) = x - A \sin(\omega t + \phi) = 0$$

Time and configuration variables can be treated on equal footings.

Hamiltonian physics in terms of partial observables

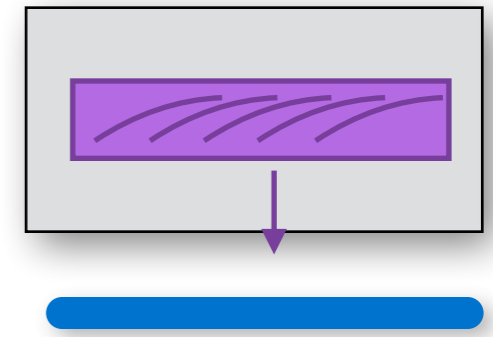
Extended configuration space: $q = (x, t) \in \mathcal{E}$

Extended phase space: $\Gamma = T_*\mathcal{E}$

Hamiltonian constraint: $C : \Gamma \rightarrow \mathbb{R}$

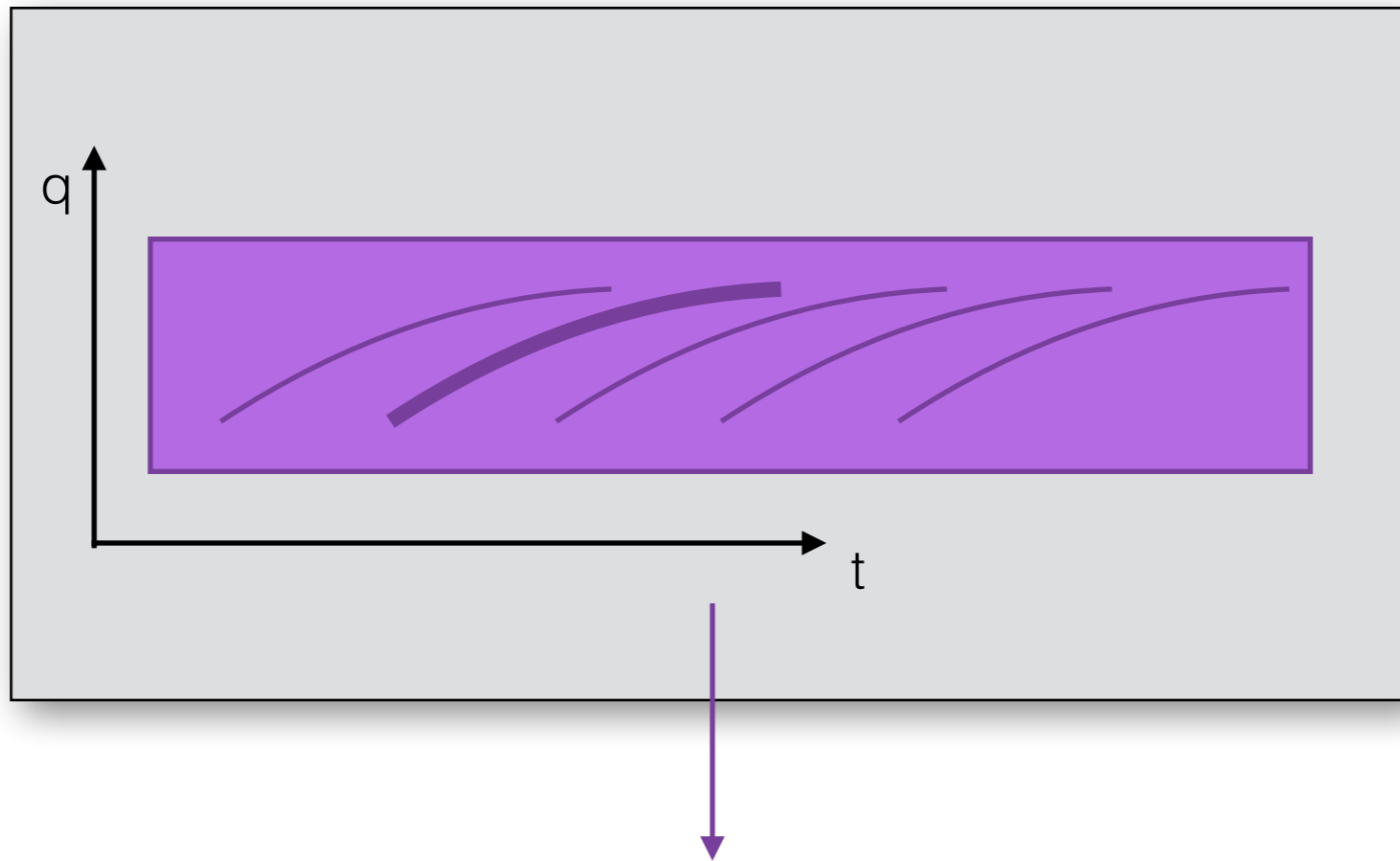
Equations of motion: $\omega(X)|_{C=0} = 0$

Cfr: Oliver



= Sean

There is no need to ever use the notion of “time” in order to have a predictive and complete theory of dynamics



This is **not** the space of the instantaneous states!
It is the space of the histories!

Hamiltonian physics in terms of partial observables

Extended configuration space: $q = (x, t) \in \mathcal{E}$

Extended phase space: $\Gamma = T_*\mathcal{E}$

Hamiltonian constraint: $C : \Gamma \rightarrow \mathbb{R}$

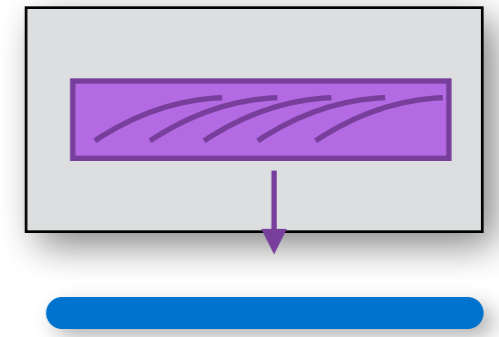
Equations of motion: $\omega(X)|_{C=0} = 0$

Cfr: Oliver

Quantum theory $\Psi(q) = \Psi(x, t)$

Wheeler deWitt equation: $C\Psi = 0$ **\neq Sean**

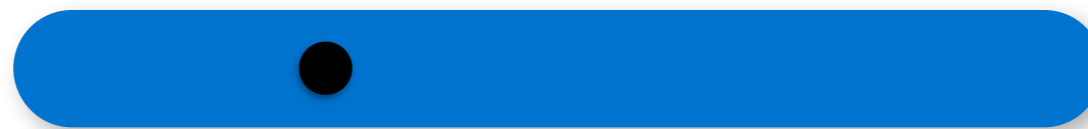
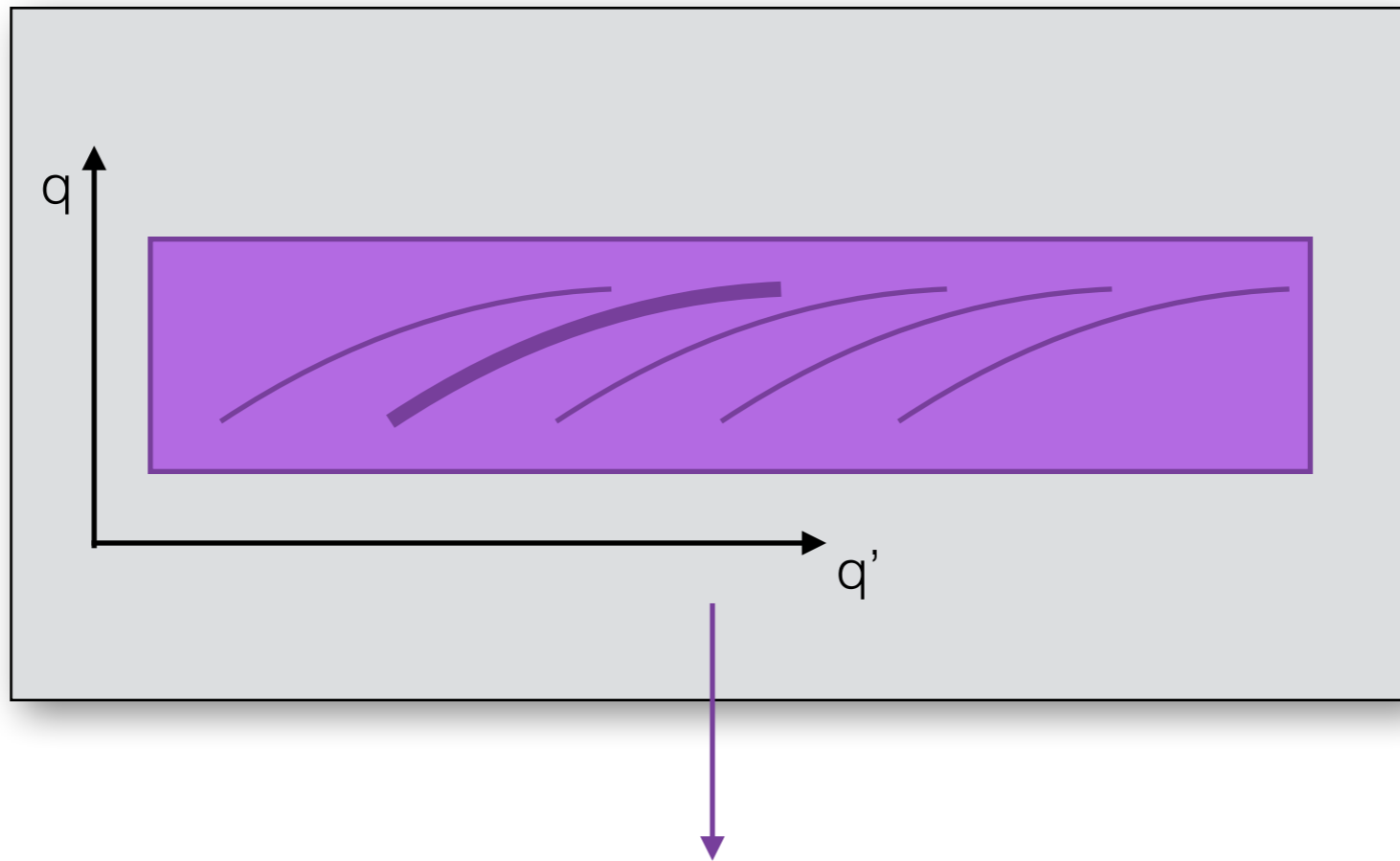
Amplitudes: $W(x, t; x' t') = \langle x, t | P | x' t' \rangle, \quad P \sim \delta(C)$
 $= \langle x | e^{-iH(t-t')} | x' \rangle$



= Sean

\neq Sean

There is no need to ever use the notion of “time” in order to have a predictive and complete theory of dynamics



This is **not** the space of the instantaneous states!
It is the space of the histories!

Classical and quantum physics without time $(x, t) \rightarrow q$ set of all partial observables

Extended configuration space: $q \in \mathcal{E}$

Extended phase space: $\Gamma = T_*\mathcal{E}$

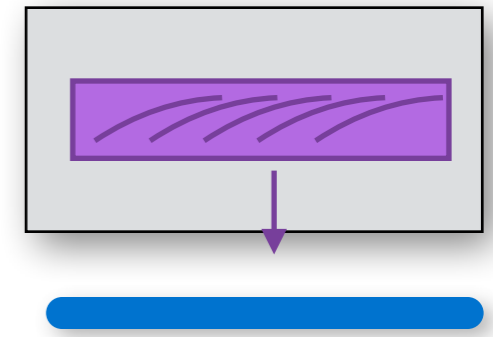
Hamiltonian constraint: $C : \Gamma \rightarrow \mathbb{R}$

Equations of motion: $\omega(X)|_{C=0} = 0$

Quantum theory $\Psi(q)$

Wheeler deWitt equation: $C\Psi = 0$

Amplitudes: $W(q; q') = \langle q | P | q' \rangle, \quad P \sim \delta(C)$



q ₁	q' ₁
q ₂	q' ₂
q ₃	q' ₃
q ₄	q' ₄
...	...

There is no need to ever use the notion of “time” in order to have a predictive and complete theory of dynamics

What needs to be diff invariant are
the **transition amplitudes**
between partial observables's
eigenstates,
not the “observables” themselves!

Dynamics describes *evolution* of dynamical variables *in time*.



Dynamics describe *relations* between partial observables

This is *possible* for any dynamical system.

It is *necessary* for a system including gravity.

Please do not confuse:

Evolving constants of motion:

Quantum evolving constants

Carlo Rovelli Physical Review D44, 1339 (1991)

$$x_T = x_T(x, p, t, p_t)$$

$$\hat{x}_T = x_T(\hat{x}, \hat{p}, \hat{t}, \hat{p}_t)$$

$$x_T(x, p, t, p_t) = \frac{1}{2}(x^2 + p^2) \sin\left(\omega(T - t) + \arctan\frac{\omega x}{p}\right)$$
$$\hat{x}_T = \frac{1}{2}(\hat{x}^2 + \hat{p}^2) \sin\left(\omega(T - \hat{t}) + \arctan\frac{\omega \hat{x}}{\hat{p}}\right)$$

Partial observables:

Quantum Gravity

Carlo Rovelli (CUP 2004)

$$f(x, p, t, p_t; x', p', t', p'_t) = 0$$

$$W(x, t; x' t') = \langle x, t | P | x' t' \rangle$$

Covariant Loop Quantum Gravity

Carlo Rovelli, Francesca Vidotto (CUP 2014)

$$W(x, t, x', t') = \left(\frac{m\omega}{h \sin(\omega(t - t'))}\right)^{\frac{1}{2}} e^{-\frac{i}{\hbar} \left[\frac{(x^2 + x'^2) \cos(\omega(t - t')) - 2xx'}{\sin^2(\omega(t - t'))} \right]}$$

Dirac observable:

Can be predicted

Partial observable:

Can be measured

What is “observable”
in GR
is **not** controversial

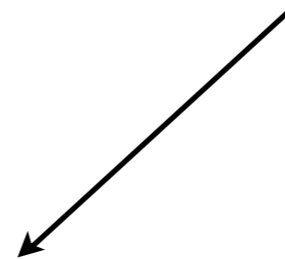
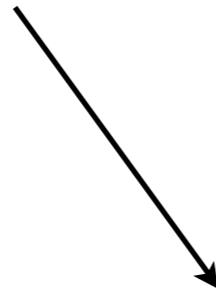
because all experimenters
agree
when they make general
relativistic observations

General relativity:

- Spacetime is the physical trajectory
the gravitational field.

Quantum theory:

- Physical trajectories do not exist.
Processes are transitions.

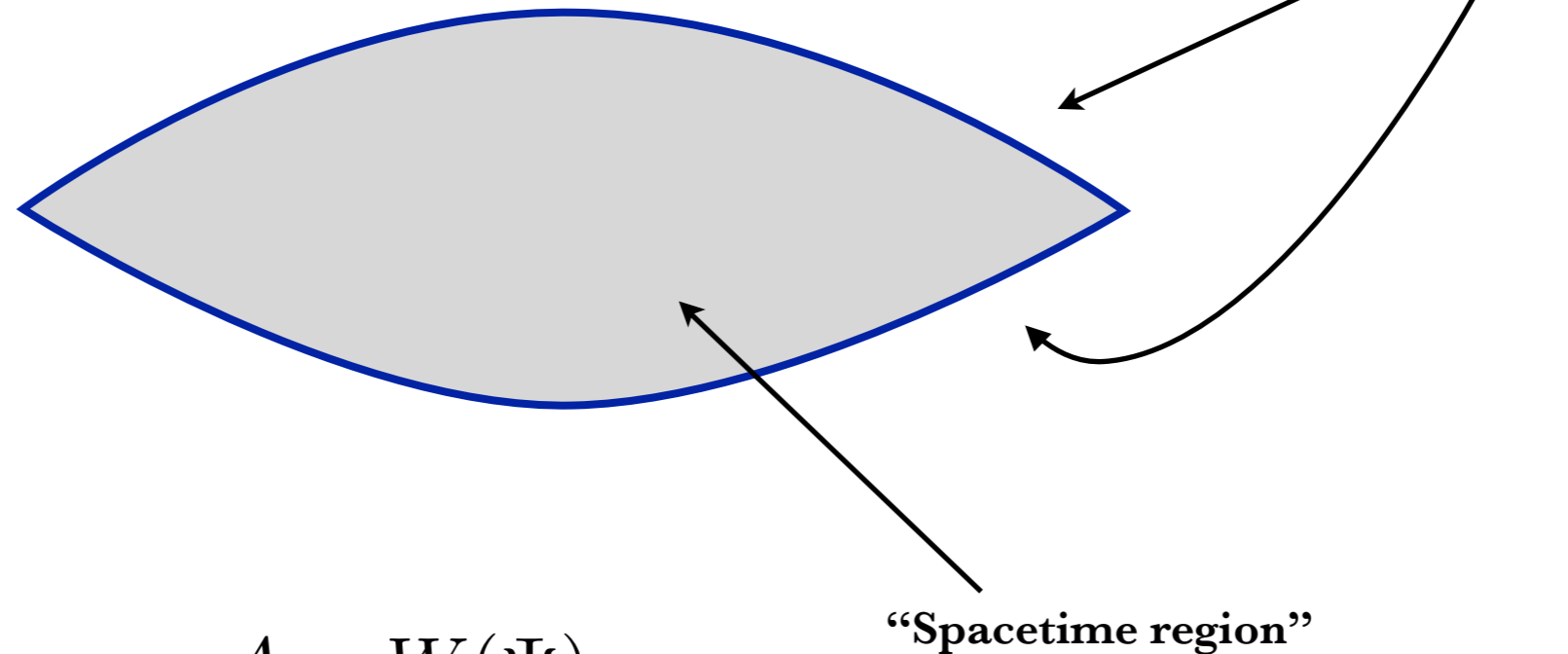


There is no spacetime in quantum gravity, and
in particular there is “no time”

A **process**, and its amplitude

Boundary state

$$\Psi = \psi_{in} \otimes \psi_{out}$$



Amplitude of the process

$$A = W(\Psi)$$

Cfr: Bianca

For example, **Loop Quantum Gravity** gives a precise mathematical definition of the state of space, the boundary observables, and the amplitude functional. (Processes, not a frozen spacetime...)

General relativistic dynamics

cfr Donald

Extended configuration space: $q \in \mathcal{E}$ space of 3-geometries
and other fields on a boundary

Extended phase space: $\Gamma = T_*\mathcal{E}$

Hamiltonian constraint: $C : \Gamma \rightarrow V$

Equations of motion: $\omega(X)|_{C=0} = 0$

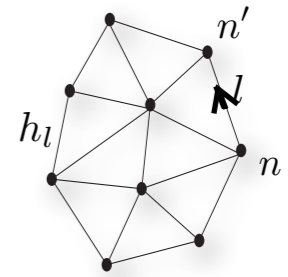
Quantum theory:

Boundary states: $\Psi \in \mathcal{H}, \quad \Psi = \psi_{in} \otimes \psi_{out}$

Transition amplitudes: $A = W(\Psi) = \langle \psi_{out} | P | \psi_{out} \rangle$

All these quantities are well defined in **Loop Quantum Gravity**

■ **Covariant loop quantum gravity. Full definition.**

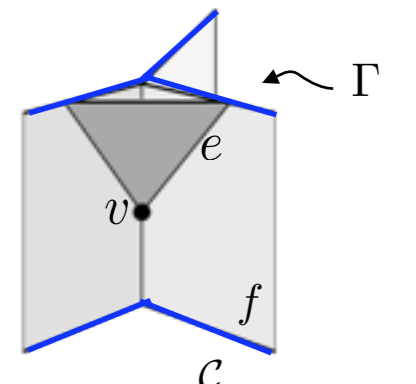
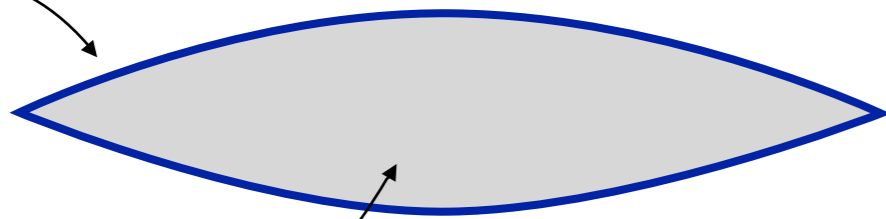


Γ spin network
(nodes, links)

Kinematics
Boundary

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L / SU(2)^N]_\Gamma \ni \psi(h_l) \quad \mathcal{H} = \lim_{\Gamma \rightarrow \infty} \mathcal{H}_\Gamma$

Operators: $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ where $L^i \psi(h) \equiv \left. \frac{d}{dt} \psi(h e^{t\tau_i}) \right|_{t=0}$



spinfoam \mathcal{C}
(vertices, edges, faces)

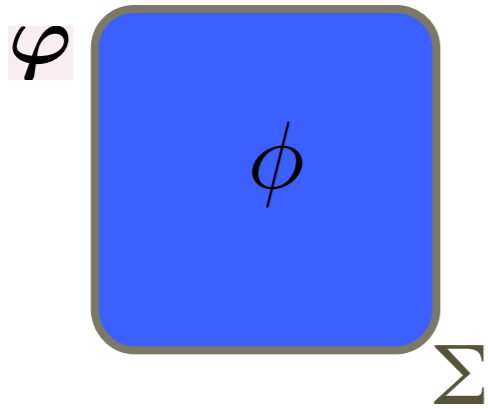
Dynamics
Bulk

Transition amplitudes $W_{\mathcal{C}}(h_l) = N_{\mathcal{C}} \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf}) \quad W = \lim_{\mathcal{C} \rightarrow \infty} W_{\mathcal{C}}$

Vertex amplitude $A(h_{vf}) = \int_{SL(2,\mathbb{C})} dg'_e \prod_f \sum_j (2j+1) D_{mn}^j(h_{vf}) D_{jmjn}^{\gamma(j+1)j}(g_e g_{e'}^{-1})$

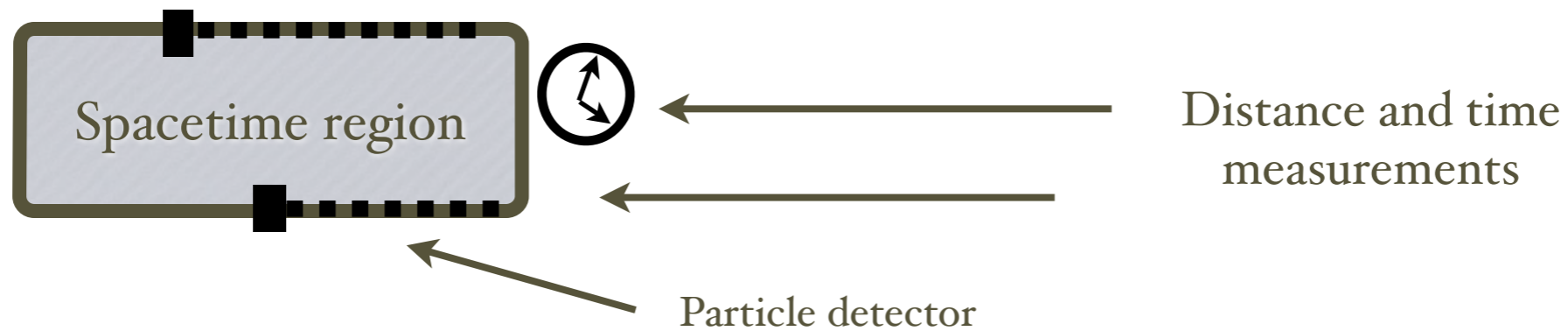
$$h_f = \prod_v h_{vf} \quad 8\pi\gamma\hbar G = 1$$

-
- The notion of “Time” is not *needed* to describe change.
 - The notion of “Time” is not *of use*, and in fact misleading, in relativistic quantum gravitational physics.



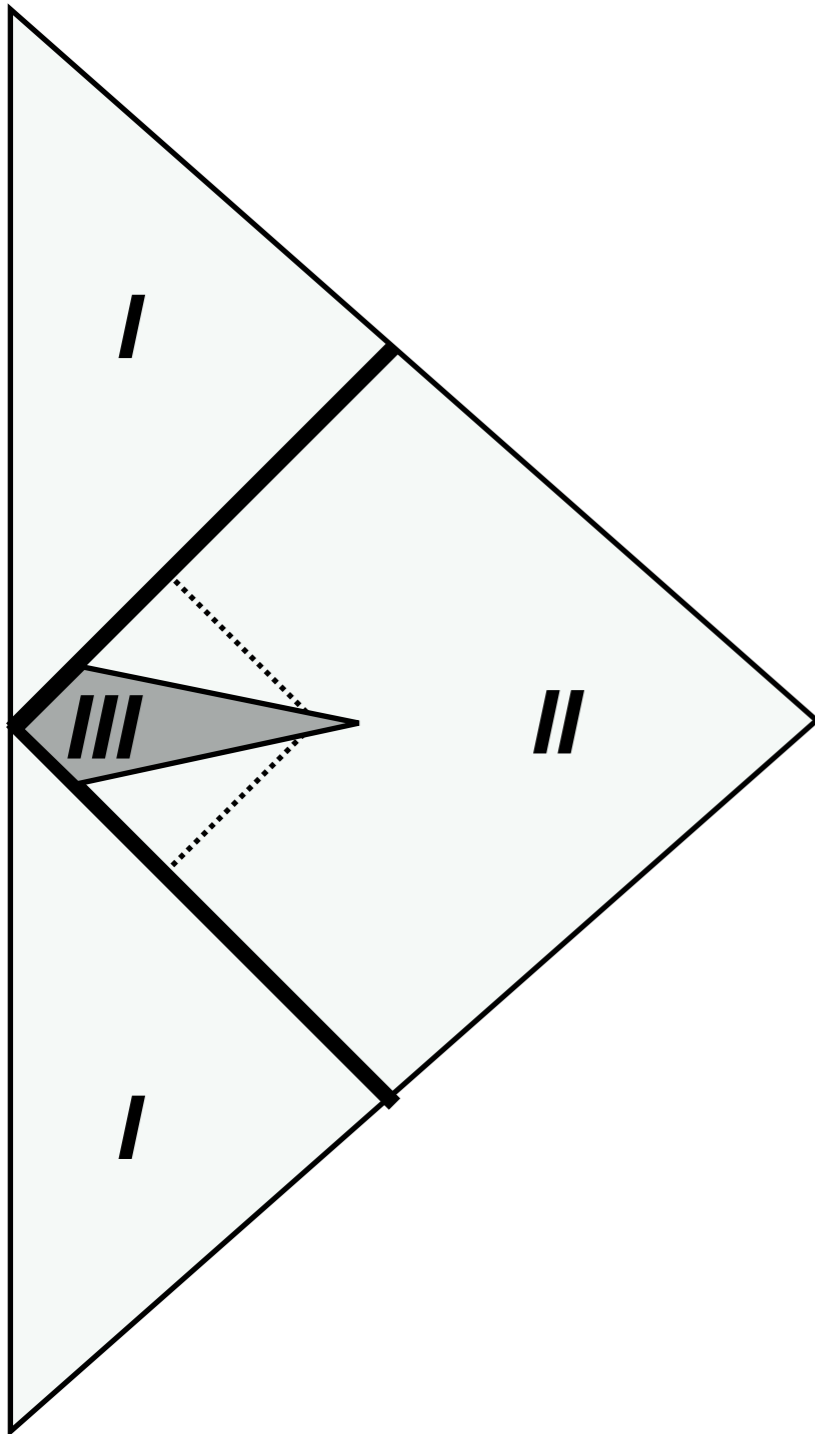
Boundary functional $W[\varphi, \Sigma] = \int_{\phi|_{\Sigma}=\varphi} D\phi e^{iS[\phi]}$

But in a generally covariant theory: $W[\varphi, \Sigma] = W[\varphi]$



In a general relativistic theory, distance and time measurements are field measurements like the other ones: they are determined by the boundary data of the problem.

A concrete application



A technical result in classical GR:

The following metric is an exact vacuum solution, plus an ingoing and outgoing shell, of the Einstein equations outside a finite spacetime region (grey).

$$ds^2 = -F(u, v)dudv + r^2(u, v)(d\theta^2 + \sin^2\theta d\phi^2)$$

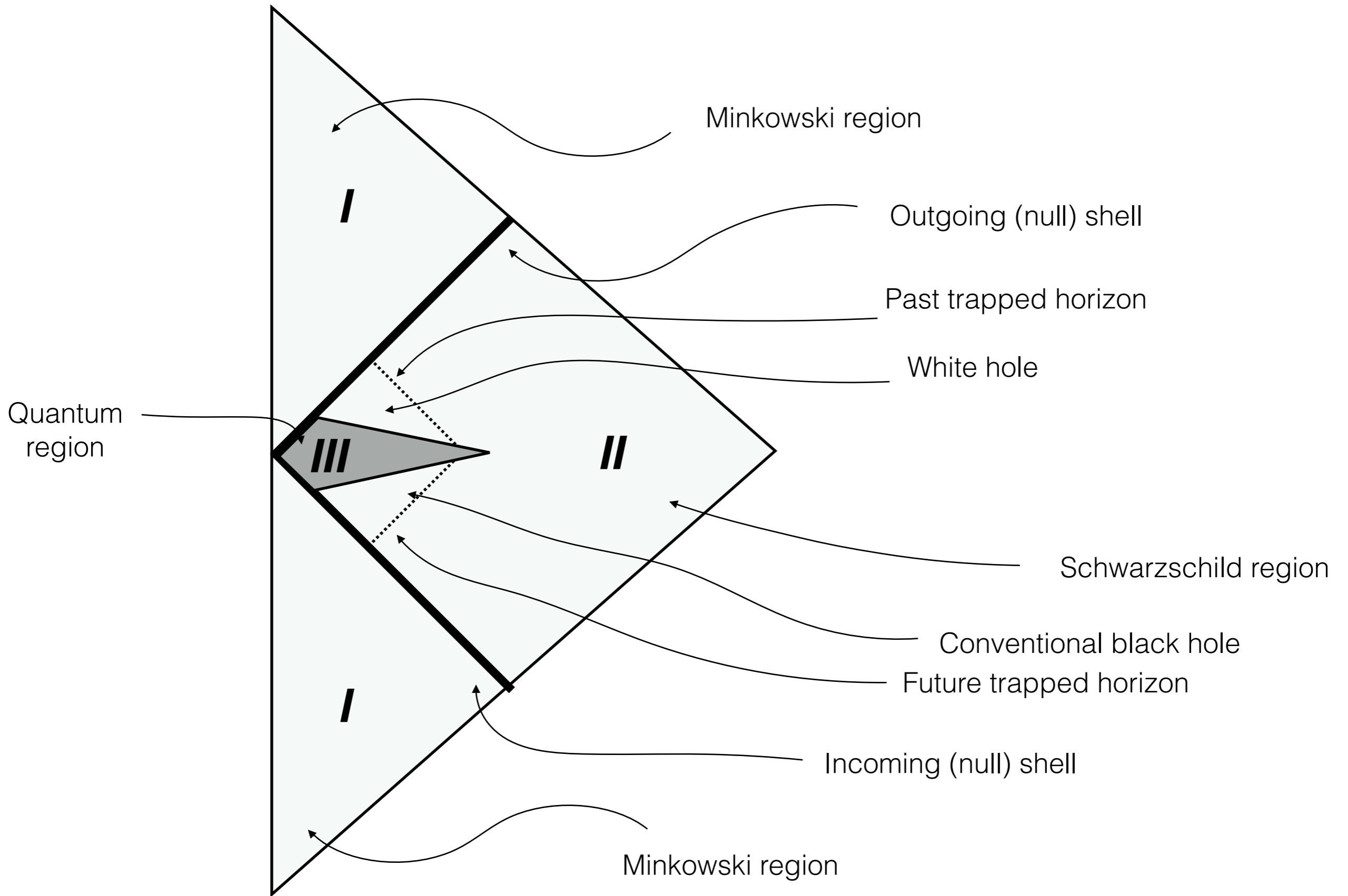
Region I $F(u_I, v_I) = 1,$ $r_I(u_I, v_I) = \frac{v_I - u_I}{2}.$
 $v_I < 0.$

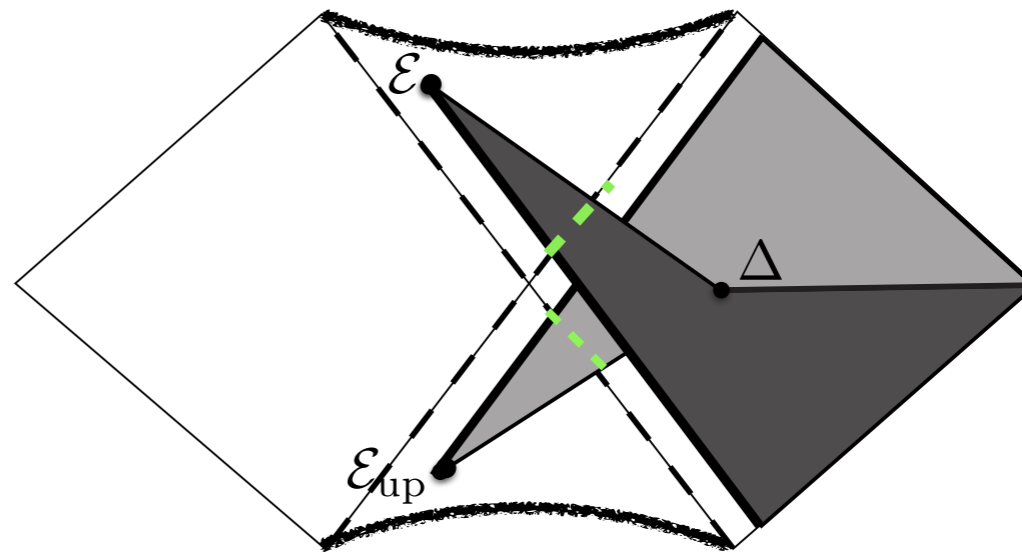
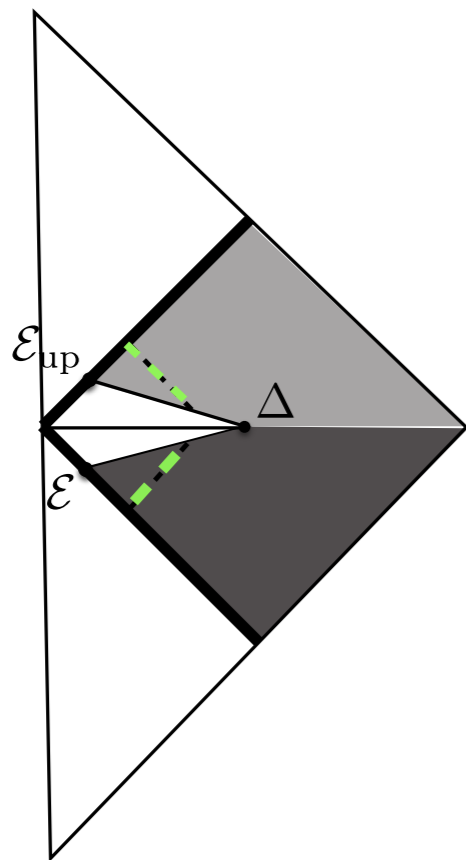
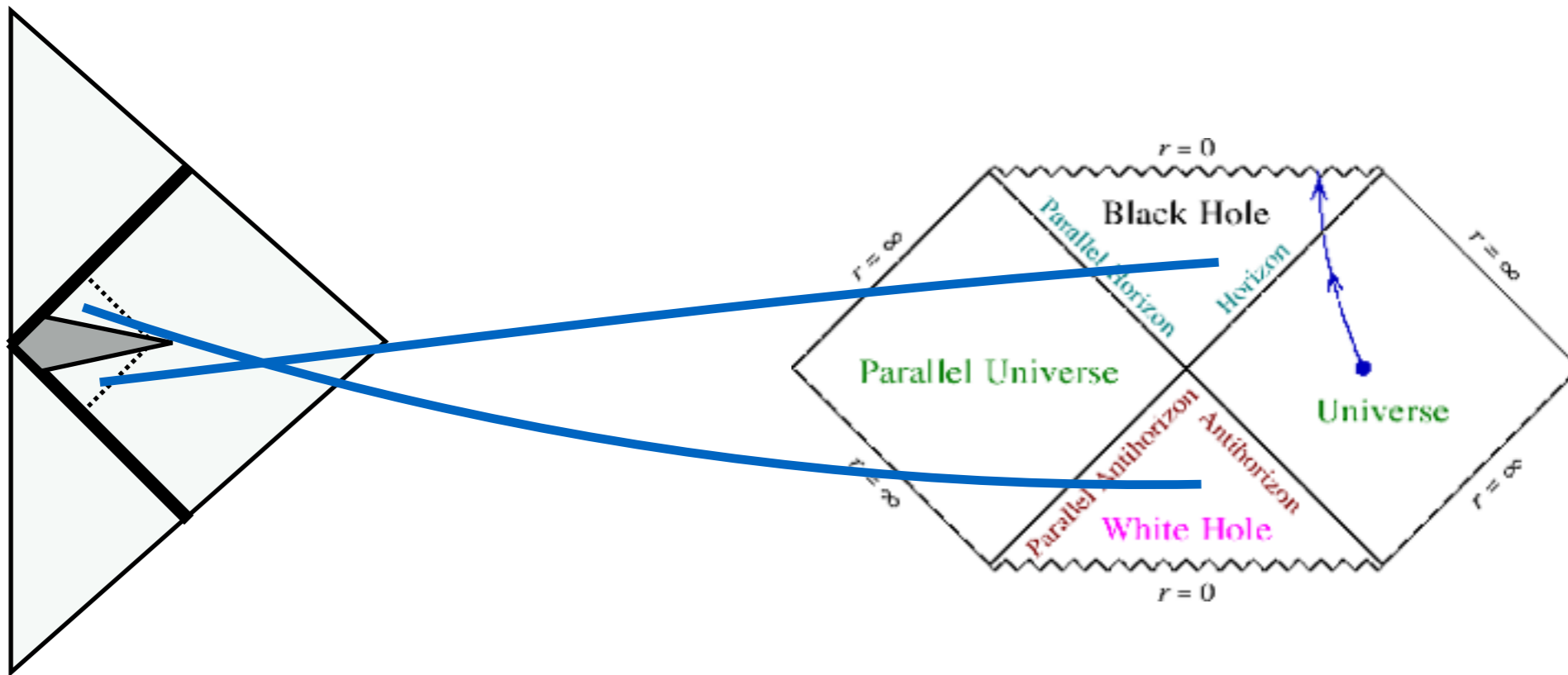
Region II $F(u, v) = \frac{32m^3}{r} e^{\frac{r}{2m}} \left(1 - \frac{r}{2m}\right) e^{\frac{r}{2m}} = uv.$

Matching: $r_I(u_I, v_I) = r(u, v) \rightarrow u(u_I) = \frac{1}{v_o} \left(1 + \frac{u_I}{4m}\right) e^{\frac{u_I}{4m}}.$

Region III $F(u_q, v_q) = \frac{32m^3}{r_q} e^{\frac{r_q}{2m}},$ $r_q = v_q - u_q.$

The metric is determined by three constants: m, ϵ, δ





The Fingers Crossed

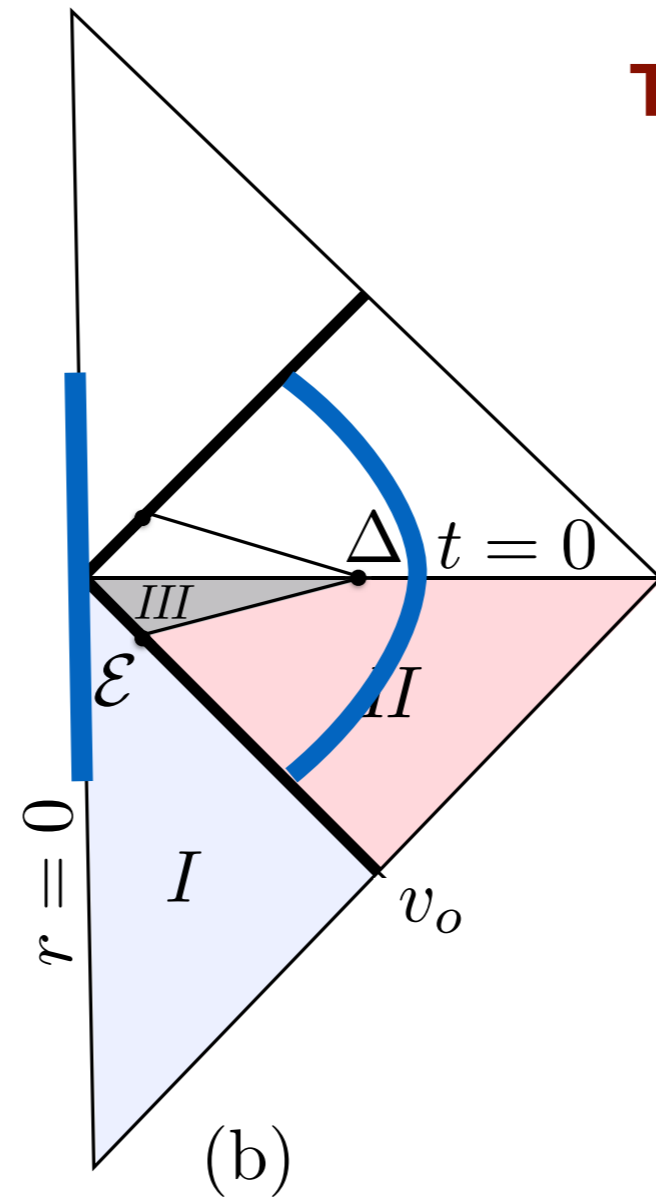
Time dilation

$$T_R = \sqrt{1 - \frac{2m}{R}} (2(R - 2m) - 4m \log v_o)$$

T: bounce time (very large)

$\tau_{internal} \sim m \sim 1ms$

$\tau_{external} \sim m^2 \sim 10^9 years$



“A black hole is a short cut to the future”

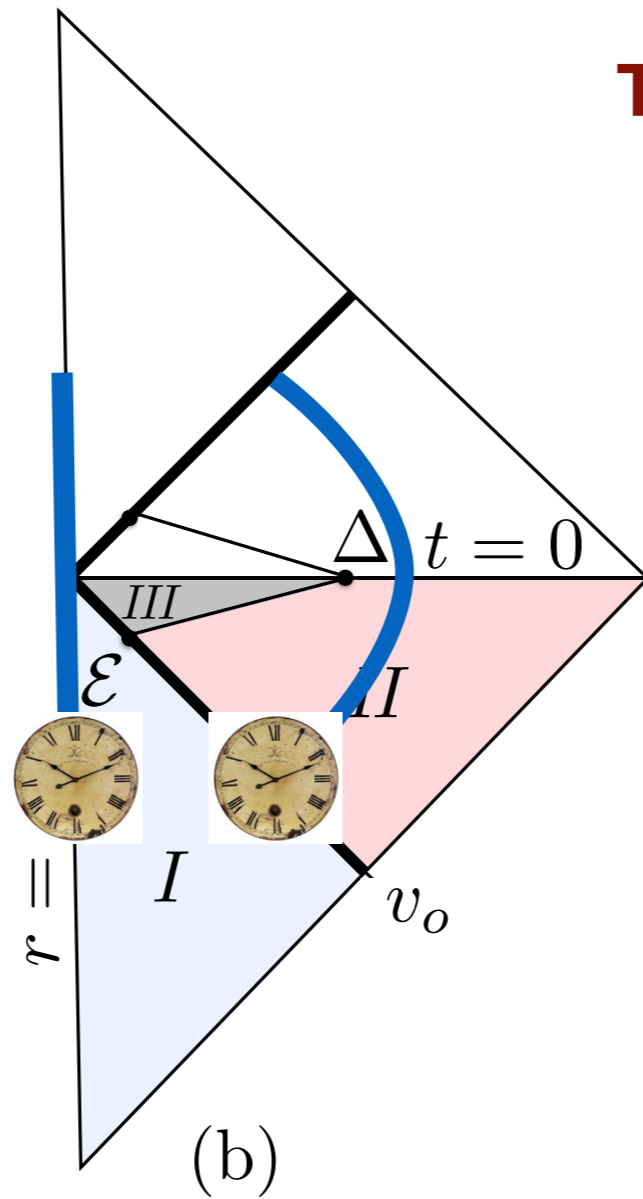
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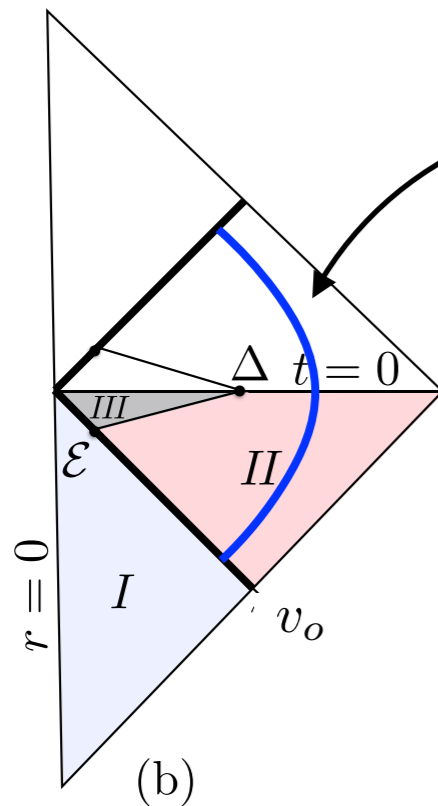
$\tau_{external} \sim m^2 \sim 10^9 years$



“A black hole is a short cut to the future”

The external metric is determined by two constants:

- m is the mass of the collapsing shell.
- v_o determines the time between the collapse and the explosion

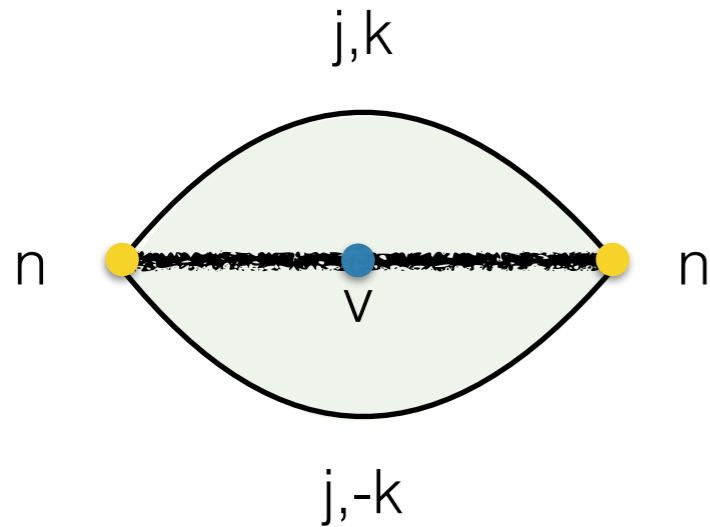
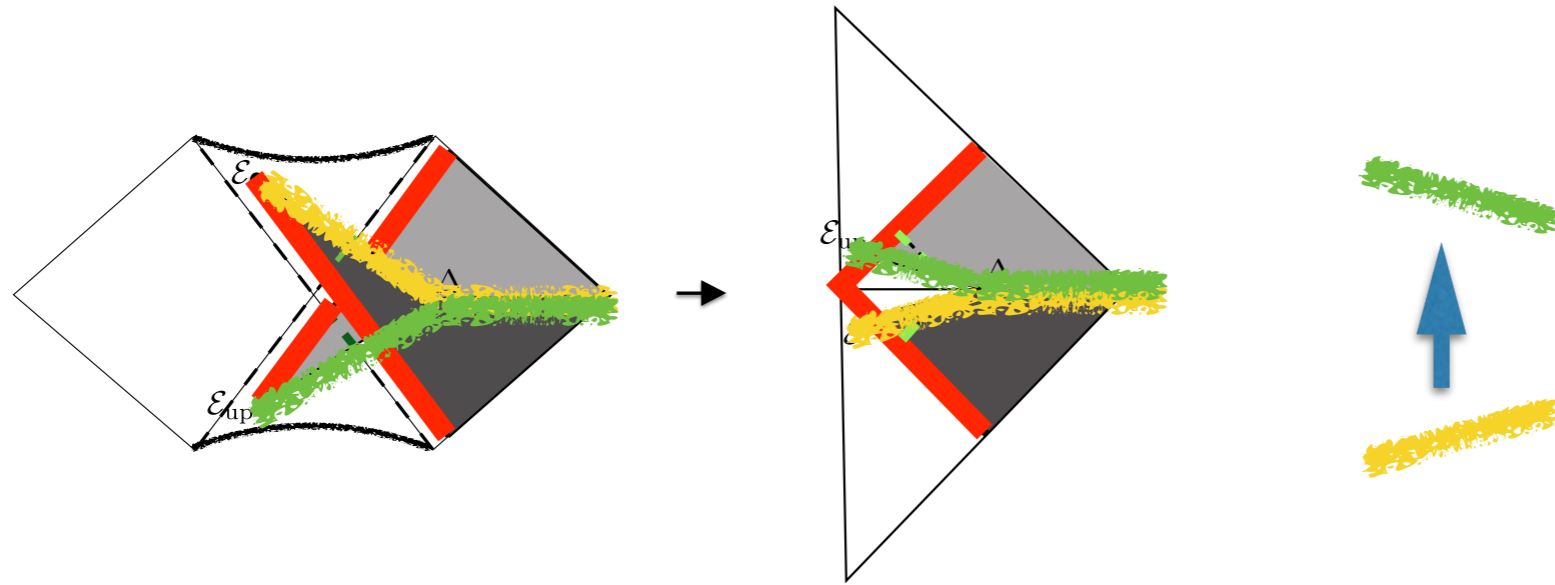


$$T_R = \sqrt{1 - \frac{2m}{R}} (2(R - 2m) - 4m \log v_o)$$

$$T = -4m \log v_o$$

What determines $T(m)$?

■ Covariant loop quantum gravity. Calculation of $T(m)$.



$$A(j, k) = \int_{SL(2C)} dg \int_{SU2} dh_- \int_{SU2} dh_+ \sum_{j_+ j_-} e^{-(j_+ - j_-)^2} e^{-(j_+ - j_-)^2} \text{Tr}_{j_+} [e^{k\sigma_3} Y^\dagger g Y] \text{Tr}_{j_-} [e^{k\sigma_3} Y^\dagger g Y]$$

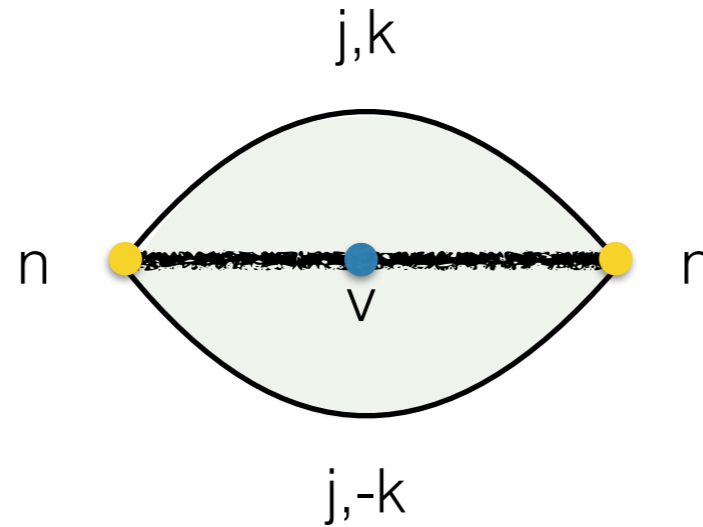
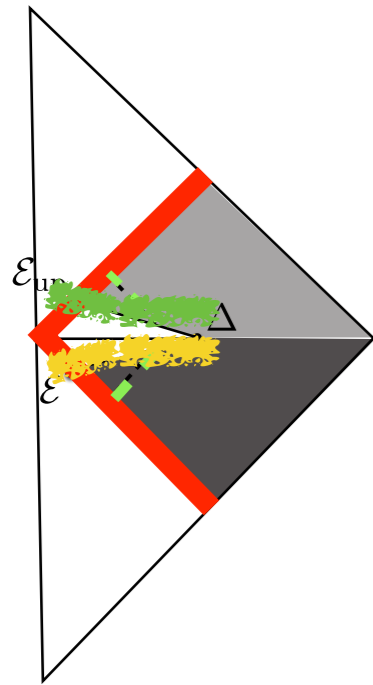
$$|A(j, k)|^2 \sim 1$$

relation j-k

relation m-time

$$\longrightarrow T(m)$$

Black to white hole tunnelling and Planck stars



Planck stars

Carlo Rovelli, Francesca Vidotto

Int.J.Mod.Phys. D23 (2014) 12, 1442026

Planck star phenomenology

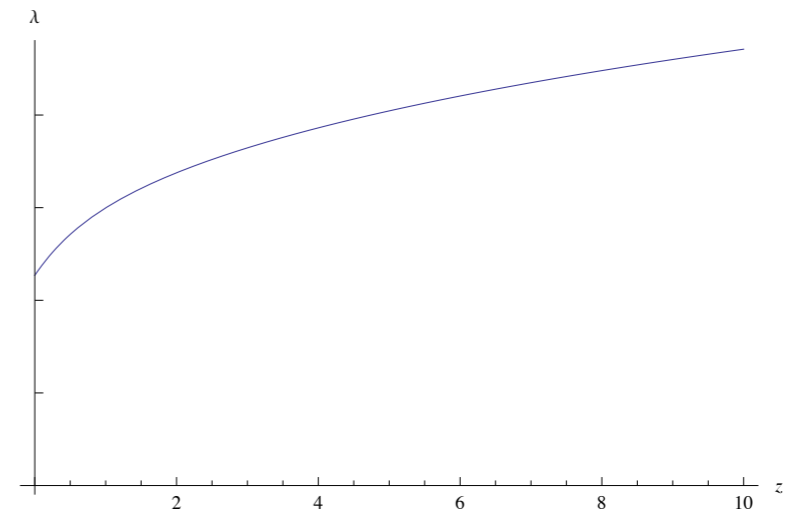
Aurelien Barrau, CR.

Phys.Lett. B739 (2014) 405

For $T \sim m^3$ primordial black hole give signals in the cosmic ray spectrum

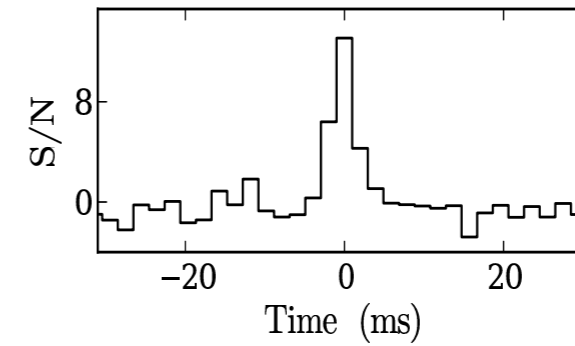
For $T \sim m^2$ primordial black hole give signals in the radio: Fast Radio Bursts?

Signature: Frequency distance dependence



Detectable?

Already detected?



For $T \sim m^2$ primordial black hole give signals in the radio: Fast Radio Bursts?

Fast Radio Bursts and White Hole Signals
Aurélien Barrau, Carlo Rovelli, Francesca Vidotto.
Phys.Rev. D90 (2014) 12, 127503



Fast Radio Bursts

- Duration: ~ milliseconds
- Frequency: 1.3 GHz
- Observed at: Parkes, Arecibo
- Origin: Likely extragalactic
- Estimated emitted power: 10^{38} erg
- Physical source: [unknown](#).

-
- In quantum gravity we need transition amplitudes between boundary states.
 - States may (or may not) depend on quantities that may happen to have an interpretation in terms of some clock time.

What about the “temporal” aspect of time?

(time “flows”, special direction, memory ..)

These are all **thermodynamical** and **statistical** phenomena. They do not pertain to the fundamental equations. They pertain to the the domain of incomplete information, coarse graining, special observers.

Statistical mechanics of gravity and the thermodynamical origin of time

[Carlo Rovelli](#) *Class.Quant.Grav.* 10 (1993) 1549-1566.

Von Neumann algebra automorphisms and time thermodynamics relation in general covariant quantum theories

[A. Connes](#), [Carlo Rovelli](#), *Class.Quant.Grav.* 11 (1994) 2899-2918

General relativistic statistical mechanics

[Carlo Rovelli](#), *Phys.Rev. D*87 (2013) 8, 084055

Why do we remember the past and not the future? The 'time oriented coarse graining' hypothesis

[Carlo Rovelli](#), arXiv:1407.3384

Statistical mechanics of reparametrization invariant systems. Takes Three to Tango

[Thibaut Josset](#), [Goffredo Chirco](#), [Carlo Rovelli](#), arXiv:1503.08725

Conclusion

- i. **The notion of “time” is not needed for mechanics:** rather than describing how physical variables (“**partial observables**”) change *in time* we can describe how they *change with respect to one another*.
- ii. This **relational form of dynamics is necessary** when dealing with the dynamics of the **gravitational field**, because Newtonian space and times are aspects of this field.
- iii. **In Quantum Gravity, the fundamental equations do not involve time.** The theory gives **transition amplitudes** for boundary states. This allows us doing standard physics.
- iv. The “temporal” and common sense aspects of the non relativistic time variable pertain to **thermodynamics** and **statistical** mechanics, not to the fundamental theory.

The mistake:

There is change

Therefore there is a preferred
time variable

Change?

CM: Relations between
partial observables

QM: Transition amplitudes!

Time variable?

Forget!

