Facets of Time in Physics

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Overview

A Panorama View on Time

Time is a topic in physics, philosophy and psychology/biology ...

Augustine's confession

What then, is time? - If no-one asks me what time is, I do know what it is. If I wish to explain it to him who asks, I do not know.

Physics should be the key to explain what time is;
textbooks and articles are rich with letters denoting time: t, x^0, T, τ

Brian Greene: YouTube video "The Illusion of Time", 2012

... ask physicists what time actually is,
... and the answer might shock you: They have no idea.

Physicists can't seem to find the time - literally. Can philosophers help ?

From Aristoteles, Newton, Leibniz to Einstein and ...

... and to Barbour, Rovelli, Smolin, Gryb and Thébault, ...

Contemporary slogans about time

- Julian Barbour: "The End of Time"
- Lee Smolin: "Time Reborn"
- Carlo Rovelli: "Forget Time"
- Sean Gryb and Karim Thébault: "Time Remains"

Do they all refer to the same "time"?
What do they understand by the death and/or resurrection of time?

The Problem(s) of Time in (Quantum) Gravity

- "frozen time"
- the problem of finding "internal" coordinates
- the multiple-time problem
- timeless-ness of the WDW equations
- observables as constants of motion
- ...
- the clash between the external time in quantum theory and the dynamical time in general relativity

Modeling Time

Mathematical characterization of time

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in terms of a topology, order relation, metric
on a one-dimensional manifold

Carlo Rovelli (1995)
"Analysis of the Distinct Meanings of the Notion of 'Time' in Different Physical Theories"

Peter Kroes (1985) *Time: its Structure and Role in Physical Theories*

Mathematical characterization of time

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in terms of symmetry restrictions

- Galilei invariance
  the model of time in classical mechanics is not simply $\mathbb{R}$,
  but it is a one-dimensional manifold with a metric;
  without an ordering $<$ and without a preferred 'now'.
- Lorentz invariance
  time is no longer represented as a single line,
  but a three-parameter family of lines $t_v = \gamma(t + \vec{v}\vec{x}/c^2)$
  i.e the straight lines filling the light cone.
- Diffeomorphism invariance
  renders impossible to single out a preferred notion of time

(For more details, see Rovelli's article.)

Mathematical characterization of time

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in terms of sets, forks, lumps

- discrete/quantized time
- forking time
- beginning/end of time
- cyclic time

Time and space: are they independent notions?

- According to the Lorentz transformations,
  distances and time intervals do not transform independently
- Since Minkowski we are no longer talking about time and space,
  but about spacetime.
- The CPT theorem combines
  time reversal, space reflection and charge conjugation
  and establishes (under rather mild assumptions) that the laws of fundamental physics are invariant under CPT transformations

$\curvearrowright$ Chronometry needs Geometry

What distinguishes time and space?

Simple Answer: The Minus-Sign

- $ds^2 = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2$
  in a coordinate-independent way: the signature of the metric allows to distinguish timelike and spacelike directions
- the minus-sign implies causality
- due to the minus sign the relativistic field equations are of hyperbolic nature. Only for these, one may specify initial data at a particular time and observe how the data propagate into the future.

Why are we talking about the *problem of time*,

but not about the *problem of space*?

Why only one time dimension?

- Instability of Particles
  problems with energy-momentum conservation allowing for non-observed decays

  J. Dorling (1970), "The Dimensionality of Time",
- Insufficient Predictabiliy
  in the sense of either/or a well-posed initial-value and boundary-value problem.

  Max Tegmark (1997), "On the dimensionality of spacetime",
- Undefined Particle Spectrum:
  for more than one time dimension the irreducible unitary representations of the Poincaré group are infinite dimensional

  H. van Dam, Y. Jack Ng (2001)
  "Why 3+1 metric rather than 4+0 or 2+2?",

Is time "quantized"/discrete?

no experimental evidence in favor, no principle objections

various completely independent viewpoints, e.g.

- R. Lévi (1927)
  "Hypothese de l'atome de temps (chronon)"
- H. Snyder (1947):
  only the Lorentz group is compatible with discrete spacetime
  C.N. Yang (1947):
  de Sitter spacetime is compatible with discrete spacetime
- P. Caldirola (1980);
  see also: Farias/Recami (2007) ... consequences for quantum mechanics

String theory and LQG assume/find
coarse-grained structures on microscopic scales.
discrete spectrum of time as related to cosmological constant

$\Longrightarrow$ talk by Francesca Vidotto

Time in Classical Mechanics

Newtonian form of mechanics

- absolute time and space
  externally given and unaffected by any material agency
- absolute notion of simultaneity
- Newton's three laws
  hold in inertial frames.
  These are at rest in absolute space or move uniformly along straight lines and are interrelated by the Galilei transformations

$$\mathbf{x} \to \mathbf{x}' = \mathbf{x} + \mathbf{v}t + \mathbf{a} \qquad\qquad t \to t' = t + \tau$$

  with constants $\mathbf{v}, \mathbf{a}, \tau$.
- Equations of motion are second-order differential equations:
  Past and future behaviour of the system is determined uniquely
  by specifying the initial positions and velocities of all mass points.

Objections by Leibniz, Mach, Poincaré, Reichenbach ...

Dynamics derived from variational principles

- Fermat's principle of least time
- The principle of least action by Maupertuis
- Least action principle due to Euler and Lagrange

$$S[q^k] = \int_{t_1}^{t_2} L(q^k, \dot{q}^k, t)dt = \int (p_k \dot{q}^k - H(q,p,t))dt$$

with either the Lagrangian L or with the Hamiltonian H

where $p_k = \frac{\partial L}{\partial \dot{q}^k}$

$$0 \stackrel{!}{=} \delta S \curvearrowright \begin{cases} \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^k} - \frac{\partial L}{\partial q^k} = 0 & \text{Lagrange equations ,} \\ d\dot{q}^k = \frac{\partial H}{\partial p_k}, \ d\dot{p}_k = -\frac{\partial H}{\partial q^k} & \text{Hamilton equations} \end{cases}$$

(assuming that the δq^k vanish at the boundary t_1 and t_2).

Hamilton-Jacobi

Hamilton's *principal function* $\bar{S}$ obeys the Hamilton-Jacobi equation

$$H(q^k, \frac{\partial \bar{S}}{\partial q^k}, t) + \frac{\partial \bar{S}}{\partial t} = 0$$

If $H = \frac{1}{2m} p^2 + U(q,t)$

$$\frac{1}{2m}(\nabla \bar{S})^2 + U + \frac{\partial \bar{S}}{\partial t} = 0.$$

Hamilton-Jacobi equation and Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla\psi + U\psi.$$

ansatz $\psi = \psi_0 \, e^{iS/\hbar}$

$$\curvearrowright \quad \frac{1}{2m}(\nabla S)^2 + U + \frac{\partial S}{\partial t} = \frac{i\hbar}{2m}\nabla^2 S.$$

For $\hbar \to 0$ the right hand side drops out

the phase of the wave function obeys the HJ-equation.

Mechanics without (fixed background) time

- Jacobi formulation of classical mechanics
- Parametrized mechanics
- Relational formulations

Jacobi formulation of classical mechanics I

For

$$L = \frac{1}{2} m_{ik}(q)\dot{q}^i \dot{q}^k - V(q)$$

the Lagrange equations are geodesics of

$$ds^2 = 2\,[E - V(q)]m_{ik}(q)\,dq^i dq^k .$$

Here E is the conserved energy.
Time can be defined from

$$\frac{ds}{dt} = \sqrt{2\,[E - V(q)]m_{ik}(q)\,\frac{dq^i}{dt}\frac{dq^k}{dt}} = 2[E - V(q)]$$

$$dt = \frac{ds}{2[E - V(q)]} = \sqrt{\frac{m_{ik}(q)\,dq^i dq^k}{2[E - V(q)]}}$$

This is what is known in celestial mechanics as *ephemeris* time.

Jacobi formulation of classical mechanics II

The equations of motion can be derived from the action

$$S[q] = \int_{\sigma_A}^{\sigma_B} d\sigma \sqrt{m_{ik}(q)\, \dot{q}^i \dot{q}^k} \sqrt{2\,[E - V(q)]}, \qquad \dot{q} := dq/d\sigma.$$

Jacobi's action is invariant under changes of the parametrization σ. Thus it describes a system with a phase-space constraint:

$$\mathcal{H}(q,p) := \frac{1}{2} m^{ik} p_i p_k + V(q) - E \approx 0$$

Since the canonical Hamiltonian vanishes, the total Hamiltonian is proportional to the constraint

$$H = N\mathcal{H}(q,p)$$

Here N is a multiplier - or - the lapse function: $dt = N d\sigma$.

The Jacobi formulation received a renaissance in the context of QG
- J. David Brown, James W. York, PRD40, 3312 (1989)
- S. Gryb, PRD81, 044035 (2010)

Parametrized Mechanics I

Classical mechanics can be rendered into a form in which its action becomes reparametrization invariant.

- introducing the time parameter as a further 'configuration' variable
  Start from a Lagrangian for a system with degrees of freedom $q_k(t)$.
  Instead of t, choose another parameter $\lambda(t)$ $\left(\text{with } \frac{d\lambda}{dt} > 0\right)$.
  Introduce a further configuration variable $q_0 := t$. Then

$$S[q_\alpha] = \int_{\lambda_1}^{\lambda_2} L_\lambda(q_\alpha, q'_\alpha) d\lambda \qquad L_\lambda(q_\alpha, q'_\alpha) = L\left(q_\alpha, \frac{q'_k}{q'_0}\right) q'_0.$$

- re-formulating classical mechanics as a constrained system
  Due to reparametrization invariance there is a constraint

$$\mathcal{H} := p^0 + H(q,p) \approx 0.$$

  The total Hamiltonian is $H_\lambda = N\mathcal{H}$ with a multiplier N.
  From the equations of motion one gets $dt = N d\lambda$. The arbitrariness of N reflects the arbitrariness of choosing the function $\lambda(t)$.

Parametrized Mechanics II

- deparametrizing the theory by a choice of a time-variable
  Although no "time" appears explicitly, there is a natural candidate for time, namely $T = q_0$, the canonically conjugate to p_0.
  The constraint can be solved for p_0: $p_0 = -H$.
  H is recovered as the true or reduced Hamiltonian depending on the true degrees of freedom only
- Dirac quantization leading to Schrödinger equation
  The constraint leads by Dirac's prescription for the quantization of constrained systems directly to the Schrödinger equation

$$\hat{\mathcal{H}}|\Psi\rangle = \left[\hat{p}^0 + H(\hat{q}, \hat{p})\right]|\Psi\rangle = 0$$

Parametrized Mechanics III

evolving constants of motion

general understanding:
In a theory with constraints, *(Dirac) observables* $\mathcal{O}$ must have vanishing Poisson brackets with the symmetry generators:

$$0 \overset{!}{\approx} \{\mathcal{O}, \mathcal{H}\} \quad \Longrightarrow \quad \{\mathcal{O}, H\} \approx 0$$

$\curvearrowright$ Observables do not evolve in the parameter λ. (PoT raises its head)

BUT: the parameter λ has no physical meaning;

idea: deparametrize by introducing "time" T

(for the most simple example $H = p^2/2m$) define

$$Q(T) := q + \frac{p}{m}(T - q_o)$$

This is an observable and an evolving constant of motion.

Relational formulations I

realizing Mach'ian ideas

Barbour (1999):

> "..., what should be known as Hofmann-Reissner-Schrödinger theory is now often referred to as Barbour-Bertotti theory."

- "Poincaré's defect":
  Specifying initial data for the distances r_{ij} and their time derivatives does not determine the dynamics of the system completely.
- Leibniz group replacing the Galilei group
  If the laws of mechanics only depend on relative distances and if their is no absolute time, they need to be invariant under

  $$\vec{x} \;\rightarrow\; \vec{x} + \mathbf{A}(\lambda)\vec{x} + \vec{g}(\lambda) \qquad\qquad \lambda \rightarrow f(\lambda),$$

  where $\mathbf{A}$ is an orthogonal matrix, and $\vec{g}$ and f are arbitrary functions (with $df/d\lambda > 0$).

Relational formulations II

- Barbour/Bertotti (1977/1982):
  Any action $S = \int d\lambda\, \mathcal{L}(\mathbf{r}_i, \mathbf{r}'_i)$ with Leibniz symmetry
  is a constrained system and entails (seven) Noether identities.
- Barbour-Bertotti model - an example

$$\mathcal{L} = \sqrt{-V\,T} \qquad \text{with} \qquad V := \sum_{i<j} \frac{m_i m_j}{r_{ij}} \qquad T = \Big(\sum_{i<j} \frac{m_i m_j}{r_{ij}} r'_{ij} \Big)^{1/2}$$

- The total momentum and angular momentum
  vanish by the constraints.
- In choosing the gauge $T + V = 0$, the τ-reparametrization is broken;
  this defines time
  and Newton's theory is recovered.
- Further developments (Barbour et al.):
  best-matching $\rightarrow$ conformal superspace $\rightarrow$ "old" shape dynamics

Time in Special Relativity

Simultaneity

- The starting point of SRT was not Lorentz transformations, ..., but "Simultaneity"

  Zur Elektrodynamik bewegter Körper

  I. Kinematischer Teil §1. Definition der Gleichzeitigkeit

- simultaneity is prior to time
  and serves to define time[1]

  Die "Zeit" eines Ereignisses ist die mit dem Ereignis gleichzeitige Angabe einer am Orte des Ereignisses befindlichen Uhr, welche mit einer bestimmten, ruhenden Uhr, und zwar für alle Zeitbestimmungen mit der nämlichen Uhr, synchron läuft.

[1] Annalen der Physik 17 (1905)

Minkowski geometry

- Euclidean geometry and Galilei invariance:
  Galilei trafos leave invariant $d\Delta^2 = dx^2 + dy^2 + dz^2$ and dt^2
  vs. Minkowski geometry and Poincaré invariance:
  Poincaré transformations $x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$
  leave $ds^2 = c^2 dt^2$ -dx^2-dy^2-$dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ invariant
  with constant a^μ and matrices $\Lambda^\mu{}_\nu$, where $\Lambda^T \eta \Lambda = \eta$
- light cone
  allows to visualize timelike, lightlike and spacelike world lines
- proper time τ (Minkowski 1908)
  along a timelike or lightlike world line is the time as measured by a clock following that line[2]. It is a Lorentz scalar

$$\tau = \int_P d\tau = \int_P \sqrt{(c^2 dt^2 - dx^2 - dy^2 - dz^2)/c}$$

- time dilation

[2]there are some subtleties concerning the 'clock hypothesis'.

Relativistic field theory

- Lagrangian formulation of a relativistic field theory is without bias.
- At least three Hamiltonian formulations are possible[3]
  With respect to which 'velocities' are momenta defined?
- Dirac's hypersurfaces
  (IF): $x^0 = 0$, (FF): $x^0 + x^3 = 0$, (PF):$x^2 = a^2 > 0, x^0 > 0$

P.A.M. Dirac, "Forms of Relativistic Dynamics" (1949)

There is no conclusive argument in favor of one or other of the forms. Even if it could be decided that one of them is the most convenient, this would not necessarily be the one chosen by nature, in the event that only one of them is possible for atomic systems. Thus all three forms should be studied further.

see: Leutwyler/Stern (1978) "Relativistic Dynamics on a Null Plane"

[3] in terms of group theory there are five distinguished ones; see Sundermeyer (2014).

The relativistic particle: Lagrangian and Hamiltonian

einbein form of the Lagrangian

$$L(\dot{q}^\mu, N) = \frac{1}{2N}\eta_{\mu\nu}\dot{q}^\mu\dot{q}^\nu + \frac{m^2}{2}N.$$

$q^\mu(\tau)$ are Lorentz vectors in D-dim Minkowski space. ($\dot{q}^\mu = dq^\mu/d\tau$).
Running through the Rosenfeld-Dirac-Bergmann algorithm one derives the Hamiltonian

$$H_T = \frac{1}{2}N(p^2 - m^2) + uP.$$

with the momenta

$$p_\mu := \frac{\partial L}{\partial \dot{q}^\mu} = \eta_{\mu\nu}\frac{\dot{q}^\nu}{N} \qquad\qquad P := \frac{\partial L}{\partial \dot{N}} = 0.$$

H_T is the sum of two first-class constraints $\mathcal{H} = \frac{1}{2}(p^2 - m^2)$ and P.

The relativistic particle: symmetries

The Lagrangian is (quasi)-invariant under the transformations

$$\delta\tau = \epsilon, \qquad \bar{\delta} q^{\mu} = -\dot{q}^{\mu}\epsilon, \qquad \bar{\delta} N = -\partial_{\tau}(N\epsilon).$$

The generator of infinitesimal transformations in phase-space is $G_{\zeta} = \mathcal{H}\zeta + P\dot{\zeta}$:

$$\delta_{\zeta} q^{\mu} = \{q^{\mu}, G_{\zeta}\} = \eta^{\mu\nu} p_{\nu}\zeta = \dot{q}^{\mu}\frac{\zeta}{N}, \qquad \delta_{\zeta} N = \{N, G_{\zeta}\} = \dot{\zeta}.$$

These only match with the configuration space trafos iff $\zeta = -N\epsilon$.
The parameter ζ mediating the symmetry trafo in phase space must depend on the lapse N.
This is typical for generally covariant theories, and derives from the distinction between the diffeomorphism and the Bergmann-Komar group and from the demand for projectability of the configuration-space trafos under the Legendre transformation (Pons/Salisbury/Shepley).

The relativistic particle: "Frozen time"?

Part of folklore in canonical GR:

The Hamiltonian is a gauge[4] generator.
Thus time evolution is nothing but gauge transformation.

- The vanishing of the total Hamiltonian is discussed with terms like "frozen time" or "nothing happens".
- Confusion about the seemingly double role of the constraint as a Hamiltonian and as a generator of symmetry transformations.
- Although both have the same mathematical form, the symmetry generator and the Hamiltonian act in different spaces:
  - The symmetry generator takes a complete solution and maps a point in the space of solutions to another solution.
  - The Hamiltonian takes initial data in this point (in the space of solutions) and maps these to later data in the same point.

[4] The term "gauge" is misleading anyhow; I prefer to use the wording "symmetry" for diffeomorphism-invariant theories

The relativistic particle: quantization

As there is no royal road to quantization, try one of these:

- Choose a "time" variable $T = T(q^\mu, p_\mu)$ ("internal time").
  example $T = q^0$.
  Write the constraint as $p_0^2 - \mathbf{p}^2 - m^2 = 0$ and solve this for p_0:

$$p_0^2 = \sqrt{\mathbf{p}^2 + m^2} =: H_{red}.$$

  Turn the reduced Hamilton function H_{red} into an operator, and define the quantum theory by

$$(\sqrt{\hat{\mathbf{p}}^2 + m^2})\Psi(q^k, p_k) = 0, \qquad \hat{p}_k = -i\hbar\frac{\partial}{\partial q^k}.$$

  Here is another "problem of time": different choices of an internal time might lead to unitarily non-equivalent quantum theories.
- Apply the constraints directly on the states

$$(\hat{p}^2 - m^2)\psi = 0 \qquad \hat{P}\psi = 0.$$

  This is the Klein-Gordon equation.

Special Relativity: Further (left-out) items

- Einstein-Poincaré synchronization (and beyond)
- twin 'paradox'
-
- axiomatic of space-time structures and standard clocks

Time in General Relativity

The Lagrangian formulation of GR

- Hilbert-Einstein action (+ boundary term + matter part)
  other options: formulations by Ashtekar, Plebanski, Holst, ...
  do they shed another light on "time"?
- The action is diffeomorphism invariant entailing
  - Noether identities (contracted Bianchi identities): $G^{\mu\nu}{}_{;\mu} \equiv 0$
  - Incomplete Cauchy problem:
    The four equations $G^{\mu 0} = -\kappa\, T^{\mu 0}$ do not contain information on the dynamics. These are to be imposed on the initial data as constraints.
    Only the other six field equations $G^{ij} = -\kappa\, T^{ij}$ are genuine dynamical equations. These determine the second time derivatives of only the six metric components g^{ij}.
    The second time derivatives of the four $g^{\mu 0}$ remain undetermined.
- initial data formulation (York, Lichnerowicz)
  using conformal techniques to formulate independent initial data

Time in Cosmology: geometry of the universe

Robertson-Walker metric
on large scales the universe appears homogeneous and istotropic

$$ds^2 = -N^2(t)dt^2 + e^{2\omega}(t)\frac{r^2}{1-kr^2} + r^2 d\Omega^2;$$

parameter $k = (-1, 0, +1)$ is the intrinsic curvature.
The metric is described by two functions depending on the coordinate time t: $N(t)$ (lapse) and $a(t) = e^{\omega}$ (scale factor)

Friedmann-Lemaître models
solutions of the field equations with RW-metric
with matter modeled as an ideal fluid
and allowing for a cosmological constant

Time in Cosmology: dynamics of the universe

Hubble parameter

$$H = \frac{\dot{\omega}}{N}$$

measures the rate of expansion (w.r.t the coordinate time)

deceleration parameter

$$q := -\frac{(\dot{a}/N)^{\cdot} aN}{\dot{a}^2}$$

measures the rate of change of the expansion rate.

Friedmann-Lemaître models

$$H^2 = \frac{\kappa}{3}\rho - e^{-2\omega}k + \frac{\Lambda}{3} \qquad \text{Friedmann equation}$$

$$q = H^{-2}\left[\frac{\kappa}{6}(\rho + 3P) - \frac{\Lambda}{3}\right] \qquad \text{Raychaudhuri equation}$$

General Relativity: Further (left-out) items

- Observables
-
- Time in cosmology: beginning and ending of time?
- Time travel

Time in Quantum Mechanics

Wave function evolving in time

A quantum system, described by coordinates q_α, is characterized by a wave function $\psi(q,t)$ satisfying the Schrödinger equation

$$i\hbar \frac{d}{dt}\psi(q_\alpha, t) = \hat{H}(\hat{q}, \hat{p})\psi(q, t)$$

- Quantum mechanics retains the Newtonian concept of absolute time.
- The probability to find the system in the configuration-space element $d\Omega_q$ at time t is $dP = |\psi(q,t)|^2 d\Omega_q$.
- $|\psi(q,t)|^2$ is the time component of a conserved probability current:

$$\frac{d}{dt}(\psi^*\psi) + \frac{d}{dq}\frac{i}{2}(\psi_q^*\psi - \psi^*\psi_q) = 0$$

and thus the total probability is conserved in (absolute) time.
For this Born interpretation to hold, the wave function needs to be complex-valued and the imaginary unit is mandatory

Time in QM: widespread beliefs

Time seems to be a problem in quantum mechanics:

- "Time and space play fundamental different roles in quantum mechanics"
- "(E,t)-uncertainty relation needs to be interpreted different from the one for the canonical pairs (q,p)"

Jan Hilgevoord

The problem of time in quantum mechanics has puzzled physicists right from the beginning and it is still an actively debated subject.

"Time in Quantum Mechanics: A Story of Confusion",
Stud. in Hist. Philos. of Modern Physics 36, 29-60 (2005)

Is there a Problem of Time in QM?

The confusions about time are largely due to sins of our Grand Old Men:

Dirac, Heisenberg, Bohr, Schrödinger, von Neumann, Pauli.

In their work (1925-1933) they missed to observe one or the other of

- Both classical mechanics and quantum mechanics are formulated relative to a given space-time background, coordinized by (x, y, z, t).
- The phase-space variables (q,p) in the Hamilton formulation depend on the external time t.
- Only in the one-particle case you may identify the configuration variables q_i with the Cartesian position coordinates x_i.
- It is not true that the position of a particle and the time coordinate form a relativistic four-vector.
- Energy and time do not form a canonical pair.
- In QM only the phase-space variables are turned into operators depending on t (in the Schrödinger representation).
  The background coordinates are not quantized.

Can time be measured in QM?

This would only be possible if time is an observable, but ...

The answer to the question
"Can one determine the amount of time τ a quantum particle spends in a specified region of space Ω?"
remains open - and this despite five decades of research.

Literature is full with articles concerning
traversal-time, flight-time, tunneling-time

and there are efforts to modify QM and/or define specific operators

Quantum Physics: Further (left-out) items

- Heisenberg picture, interaction picture
- Uncertainty relations
- Time and path-integrals

  $\Longrightarrow$ talk by Henrique Gomes
- Delayed choice experiments

  $\rightarrow$ erasing the past? backward causation?
- decoherent/consistent history approach
- weak measurement (Aharanov et al.)
-
- Time in quantum field theory

  $\Longrightarrow$ talk by Alex Blum

Time in Quantum Gravity

Canonical quantization in the metric approach

Three (plus x) Roads to Quantum Gravity

- *Canonical/Hamiltonian Approach*
- Strings
- Loop Quantum Gravity
- x: Causal Sets, Topos Theory, Non-Commutative Structures, ...

Three (plus x) Roads to Canonical Quantum Gravity

- *metric approach*
  Rosenfeld, Bergmann et al., Dirac, DeWitt, Arnowitt-Deser-Misner
- connection and loops
  Ashtekar
- shapes
  Gomes, Gryb, Koslowski

Which common and which different perspectives
do they shed on the notion of time ?

Canonical GR: 3+1 split

- Split of spacetime into a foliation of three-dimensional spacelike hypersurfaces (assuming that the manifold $\mathcal{M}$ be globally hyperbolic: $\mathcal{M} \simeq \mathbb{R} \times \Sigma$ with the 3dim manifold Σ.)
- Parametrize the metric as

$$g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + e_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

  with the *lapse function* N and the *shift functions* N_i, respectively.
- The lapse function determines the clock rate by which the coordinate time t measures time. It could be removed by rescaling the "time"-parameter such that $d\tau = N(t)dt$, where τ carries the meaning of proper time as measured by a co-moving observer.

Canonical GR: constraints and Hamiltonian

configuration variables: three-metric e_{ij}, lapse/shift N^α
canonically conjugate momenta: p^{ij} and P_α, resp.

The Hamiltonian density associated to the Hilbert-Einstein action turns out as a sum of constraints[5]

$$\mathcal{H} = N\mathcal{H}_\perp + N^i\mathcal{H}_i + v^\alpha P_\alpha$$

with multipliers v^α.

$$\mathcal{H}_\perp = (2\kappa)\, e^{-1/2}(p^{ij}p_{ij} - \frac{1}{2}p^2) - \frac{e^{1/2}}{2\kappa}\,{}^{(3)}R$$
$$\mathcal{H}_i = -2p_i{}^j{}_{|j}.$$

[5] if Σ is compact without a boundary; otherwise there is a surface term.

Canonical GR: Bergmann-Komar group

P. Bergmann and A. Komar (1972)

- the Hilbert-Einstein Lagrangian is quasi-invariant not only with respect to diffeomorphism but to the larger group **Q** with transformations of the form $\hat{x}^\mu = \hat{x}^\mu[x, g_{\rho\sigma}]$.
- In phase space, these generically field-dependent functionals must be restricted. Infinitesimally with $\hat{x}^\mu = x^\mu + \epsilon^\mu[x, g_{\rho\sigma}]$,:

$$\epsilon^\mu[x, g_{\rho\sigma}] = n^\mu \xi^0 + \delta^\mu_i \xi^i \qquad \text{where} \qquad \xi^\alpha = \xi^\alpha[e_{ij}, K^{ij}].$$

  Thus the transformations must depend explicitly on the lapse and shift function via $(n^\mu) = N^{-1}(1,\text{-}N^i)$.

- The transformations form the Bergmann-Komar group **BK**.
  BK is a subgroup of **Q** - as is **Diff**(**M**).
  However, **Diff**(**M**) is not a subgroup of **BK**
  The popular "gauge choice" $(N = 1, N^i = 0)$
  blurs the distinction between these symmetry groups.

Canonical GR: the dual role of the Hamiltonian

The Hamiltonian:

$$H = \int d^3x (N\mathcal{H}_\perp + N^i\mathcal{H}_i + v^\alpha P_\alpha) = N\mathcal{H}_\perp + N^i\mathcal{H}_i + v^\alpha P_\alpha$$

evolves any (field equation) solution in $\Upsilon = \{e_{ij}, N^\alpha, p^{ij}, P_\alpha\}$ from initial values to later values by Hamilton's equations $\dot{\Upsilon} = \{\Upsilon, H\}$.

The symmetry generator[6]

$$G_\xi = -(\mathcal{H}_\alpha + N^{\gamma''} C_{\alpha\gamma''}^{\beta'} P_{\beta'})\xi^\alpha - P_\alpha\dot{\xi}^\alpha$$

maps by $\delta_\xi\Upsilon = \{\Upsilon, G_\xi\}$ a solution to a neighboring solution.
Only formally (by for instance choosing $\xi^\alpha = -N^\alpha$) the Hamiltonian and the symmetry generator become mathematically identical expressions.

$\Longrightarrow$ talk by Don Salisbury
$\Longrightarrow$ talk by Brian Pitts
$\Longrightarrow$ talk by Oliver Pooley

[6] $C_{\alpha\gamma''}^{\beta'}$ are the structure functions in the Dirac constraint algebra

Quantization of reparametrization invariant systems

- introduce an "internal" time
  quantize the true degrees of freedom with the reduced Hamiltonian

  or
- impose the constraints as operators on wave functions
  and find the appropriate Hilbert space

  or
- understand time as an approximate/emergent concept
  associated with some kind of semiclassical solution

This corresponds to what *Chris Isham* calls

Time before Quantization	T{before}Q
Time after Quantization	T{after}Q
no Time	{no}T

T{before}Q: internal coordinates

Karel Kuchař:
"A Bubble-Time Canonical Formalism for Geometrodynamics" (1972)
"The Problem of Time in Canonical Quantization" (1991)

- Find a canonical transformation

$$\left(e_{ij}(\mathbf{x}), p^{ij}(\mathbf{x})\right) \rightarrow \left(X^A(\mathbf{x}), P_B(\mathbf{x}); \phi^r(\mathbf{x}), p_s(\mathbf{x})\right)$$

  call (X^A,P_B) 'internal coordinates', (ϕ^r, p_s) 'true variables'.
- Solve the diffeomorphism constraints in the form

$$P_A(\mathbf{x}) + h_A(\mathbf{x}; X^B, \phi^r, p_s] \approx 0$$

- The field equations for ϕ^r and p_s are then derivable from the reduced Hamiltonian

$$H_{red} = \int_\Sigma d^3x\, h_A(\mathbf{x}; X^B, \phi^r, p_s]\dot{X}_t^A(\mathbf{x})$$

T{before}Q: quantization

A wave functional $\Psi[\phi^r(\mathbf{x})]$ obeys

$$i\hbar\frac{\delta\Psi[\phi^r(\mathbf{x})]}{\delta X^A(\mathbf{x})} = h_A(\mathbf{x}; X^B, \hat{\phi}^r, \hat{p}_s]\Psi[\phi^r(\mathbf{x})]$$

This local Schrödinger equation is like a Tomonaga-Schwinger equation; infinitely many equations with respect to

'bubble time' $X^A(\mathbf{x})$

in the same spirit

$\Longrightarrow$ Talk by Don Salisbury

T{before}Q: problems

among others[7]

- 'multiple-choice' problem
  neither the choice of internal coordinates, nor the canonical transformation to internal coordinates and true variables is unique
- 'global-time' problem
  in general, it is not possible to find a global canonical transformation
- 'space-time' problem
  the internal coordinates must be spacetime scalars
  - how to find them ?

The canonical reduction approach could so far only be worked out in detail for specific cosmologies, cylindrical gravitational waves, linearized gravity ...

[7] ... good clocks are hard to find $\Longrightarrow$ talk by Philipp Hoehn

(Quantum) Cosmology: Hamiltonian formulation

Couple a scalar field ϕ a gravitational field.
With an appropriate rescaling of constants and fields one derives - within the RW-metric - a Lagrangian

$$\mathcal{L} = \dot{\omega} p_\omega + \dot{\phi} p_\phi - N\mathcal{H}$$

with the weakly vanishing Hamiltonian constraint

$$\mathcal{H} = \frac{1}{4} e^{-3\omega}(p_\phi^2 - p_\omega^2) + m^2 e^{3\omega}\phi^2 - k e^{\omega} + \Lambda e^{3\omega} \approx 0.$$

This constraint is equivalent to the Friedmann equation.
The Hamiltonian field equations give rise to the Raychaudhuri equation.

(Quantum) Cosmology: internal time

(1) Choose a time variable $T = g(\omega, \phi, p_\omega, p_\phi)$.
(2) Find the momentum Π conjugate to T
and the canonical transformation $(\omega, \phi, p_\omega, p_\phi) \to (T, \Pi, Q, P)$.
(3) Rewrite the Hamiltonian constraint in terms of (T, Π, Q, P).
(4) Solve the constraint as $\Pi = \Pi(T, Q, P)$ to determine the reduced Hamiltonian $\Pi = -H_{red}(Q, P, T)$.

The choice of a time variable T fixes the lapse function:
For consistency we need to have $\dot{T} = 1 = \dot{g}$.
This relates N to the "gauge" choice by:

$$1 = N\{g, \mathcal{H}\} = N \cdot F$$

The quest for positiveness of N amounts to $F(T, \Pi, Q, P) \overset{!}{>} 0$.

(Quantum) Cosmology: internal time examples (here k=0)

- $T = \omega$, *Misner time*
  the conjugate variable is $\Pi = p_\omega$.

$$\mathcal{H}_{red}(\phi, p_\phi, T) = \sqrt{p_\phi^2 + 4m^2 e^{6T}\phi^2 + 4\Lambda e^{6T}}$$

- $T = \phi$, *'matter time'*
  the conjugate variable is $\Pi = p_\phi$.

$$\mathcal{H}_{red}(\omega, p_\omega, T) = -\sqrt{p_\omega^2 - 4m^2 e^{6\omega} T^2 - 4\Lambda e^{6\omega}}$$

  Quantum cosmologies with Misner time and matter time were first investigated by Blyth/Isham (1975).
- $T = e^{-3\omega} p_\omega$, *York time* $\sim$ H
  the conjugate momentum $\sim$ volume $e^{3\omega}$.

$$\mathcal{H}_{red}(\phi, p_\phi, T) = \pm p_\phi \sqrt{T^2 - 4(m^2\phi^2 + \Lambda)}$$

  York time seems to have a privileged role in (quantum) gravity.
  It pops up in various - sometimes disparate - contexts.

(Quantum) Cosmology: conditions on internal time

- T should be a spacetime scalar
  However, Misner time[8] is a scalar only under those transformations that respect the Killing symmetries !
  In case of isotropic cosmologies the only genuine spacetime scalars are of the form $T(H, \dot{H}/N, \phi, \dot{\phi}/N)$.
- T should be monotonically increasing
  $T = \omega$ is monotonic for an ever expanding universe.
  But, generically, for an arbitrary parameter selection (k, Λ, m^2) the universe does not expand. The same is to be questioned for $T \sim H$.
- T should be globally defined
  This needs to be fulfilled in order to arrive at unitary quantum gravity (Hajicek 1986).
  Beluardi/Ferraro (1995) construct various globally defined internal time. But most of their choices violate the scalarity condition.

---

[8] and also Kuchař's internal coordinates for cylindrical waves

(Quantum) Cosmology: internal time?

Skepticism about the existence of an 'internal' time

e.g. Unruh/Wald (1989)[9]
"One reason for our skepticism is that no solution has yet emerged after over 20 years of effort."

another way out: material reference systems

[9] another 25+ years passed by

Time in Cosmology: material reference systems

- rods and clocks (Einstein)
- clocks and elastic media (DeWitt)
- ...
- dust (Brown/Kuchar) (Husain/Pawlowski)

dust as a dynamical reference system

$$S = S_G + S_M + S_D \qquad S_D = -\frac{1}{2}\int d^4x\,\sqrt{-g}\,\rho\, g^{\mu\nu}(U_\mu U_\nu + 1)$$

$$U_\mu = -\partial_\mu F + W_j \partial_\mu S^j \quad \text{with scalar fields} \quad \rho, F, W_j, S^j$$

- The full Hamiltonian constraint can be de-parametrized.
- The fields F and S^j can be used as internal variables.

For isotropic and homogeneous cosmology: $S^j = 0$
The Hamiltonian constraint is linear in the canonical momentum p_F.
Choose $T = F$ as internal time: $H_{red} = -p_F = H_G + H_M$
internal time becomes cosmic time.

T{after}Q: constraints as operators on wave functions

Wheeler-DeWitt equation(s)

$$\hat{\mathcal{H}}_\perp \Psi(e_{ij}) = 0 \qquad \hat{\mathcal{H}}_k \Psi(e_{ij}) = 0$$

Quantization takes place by ("Dirac hatting")

$$\hat{e}_{ij}\Psi = e_{ij}\Psi, \qquad \hat{p}^{ij}\Psi = -i\hbar\frac{\delta}{\delta e_{ij}}\Psi, \qquad \left[\hat{e}_{ij}(x), \hat{p}^{ij}(y)\right] = i\hbar\delta^k_{(i}\delta^l_{j)}\delta(x,y).$$

Therefore the W-DW equations are explicitly

$$\hat{\mathcal{H}}_\perp \Psi = \left[-2\kappa\,\hbar^2 G_{ijkl}\frac{\delta^2}{\delta e_{ij}\delta e_{kl}} - \frac{e^{1/2}}{2\kappa}\left({}^{(3)}R - \Lambda\right)\right]\Psi$$

$$\hat{\mathcal{H}}_k \Psi = -2D_j\, e_{kl}\frac{\hbar}{i}\frac{\delta\Psi}{\delta e_{jl}}$$

with the DeWitt metric

$$G_{ijkl} := \frac{1}{2\sqrt{e}}\left(e_{ik}e_{jl} + e_{il}e_{jk} - e_{ij}e_{kl}\right)$$

T{after}Q: 'that damned equation'

- The wave function $\Psi[\widetilde{e}_{ij}(\mathbf{x}), \varphi_A(\mathbf{x})]$ is a functional on 'superspace'. It does not depend on time.
- The W-DW equation has no $(i\frac{\partial}{\partial t})$ term
  Barbour (1993) Kiefer (1993)
- What about a Born-type probability interpretation ?
- Hilbert space problem:
  What is the inner product on the space of physical states?

Semi-classical time: time from a timeless theory (1)

An example from early quantum physics[10]

N. Mott (1931): collision of an alpha-particle (at $\mathbf{r}$) with an atom (at $\mathbf{R}$)
Start with a *time-independent* Schrödinger equation for $\Psi(\mathbf{r}, \mathbf{R})$
The ansatz

$$\Psi(\mathbf{r}, \mathbf{R}) = \psi(\mathbf{r}, \mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}}$$

leads to *time-dependent* Schrödinger equation for ψ where time is defined from the exponential through the directional derivative

$$i\frac{\partial}{\partial t} \propto i\mathbf{k} \cdot \nabla_R$$

C. Kiefer:
"The 'heavy' system acts as a 'clock' and defines the time with respect to which the 'light' system evolves."

D. Zeh: WKB time

[10] $\Longrightarrow$ talk by Henrique Gomez who interprets this case in terms of records/histories.

Semi-classical time: time from a timeless theory (2)

WKB approaches to the W-DW equation are quite common.

full treatment in Kiefer's *Quantum Gravity* and in Kiefer (1994)

example: Minisuperspace Models

- geometric and matter variables are independent of **x** (finite-dimensional systems)
- write $q^\alpha = \{e_{ij}, \varphi_A\}$.

$$S = \int dt \left[p_\alpha \dot{q}^\alpha - N(G^{\alpha\beta} p_\alpha p_\beta + U(q)) \right], \qquad U = \sqrt{e} \left[V(\varphi) -^{(3)} R) \right]$$

- W-DW equation is obtained by replacing

$$G^{\alpha\beta} p_\alpha p_\beta \longrightarrow -\hbar^2 \nabla^2 = -\frac{\hbar^2}{\sqrt{-G}} \partial_\alpha (\sqrt{-G}\, G^{\alpha\beta} \partial_\beta), \qquad \partial_\alpha := \partial / \partial q^\alpha$$

$$\curvearrowright \qquad (\hbar^2 \nabla^2 - U)\psi(q) = 0 \qquad \Longleftarrow \quad \text{W-DW}$$

Semi-classical time: time from a timeless theory (3)

- Consider the region in which ψ is oscillatory, and make the WKB ansatz

$$\psi(q) = C(q) \exp iS(q)/\hbar$$

  where S is a rapidly varying phase and C is a slowly varying.
- Expand in powers of $\hbar$
  - to order $\hbar^0$: $$G^{\alpha\beta}(\nabla_\alpha S)(\nabla_\beta S) + U(q) = 0$$ i.e. the Hamilton-Jacobi equation
  - to order $\hbar^1$: $$2\nabla S \cdot \nabla C + C\nabla^2 S = 0$$ (in the spirit of WKB a term proportional to $\nabla^2 C$ has been dropped). This expresses the conservation of a current $j^\alpha = C^2 \nabla_\alpha S$
  - to order $\hbar^2$:

"The Problem(s) of Time"

Chris Isham
"Canonical quantum gravity and the problem of time" (1973/1992)

This problem originates in the fundamental conflict between the way the concept of time is used in quantum theory, and the role it plays in a diffeomorphism-invariant theory like general relativity.

eight versions of the PoT:
Edward Anderson, "Problem of time in quantum gravity",
Ann. Phys. (Berlin) 524, 757-786 (2012)

- "frozen time"
- the problem of finding "internal" coordinates
- the multiple-time problem
- timeless-ness of the WDW equations
- observables as constants of motion
- ...

Quantum Gravity: Further (left-out) items

- Time and observables
-
- pre-BigBang cosmologies

The Direction and Flow of Time

Time reversal

- Newton's equations are invariant under $t \to -t$
- Maxwell's equations: If τ is the matrix mediating "time" reversal, and $\mathbf{T}$ the (Wigner) antiunitary symmetry operator

$$\mathbf{T} A^{\mu} \mathbf{T}^{-1} = -\tau_{\nu}{}^{\mu} A^{\nu}(\tau x) \qquad \mathbf{T} j_{\mu} \mathbf{T}^{-1} = -\tau_{\mu}{}^{\nu} j_{\nu}(\tau x)$$

- SRT: time reversal is part of the Poincaré group
- time reversal in QM (due to the Wigner theorem)[11]:

$$\Psi^{T}(x,t) = \Psi^{*}(x,-t)$$

- time reversal in GR: which time to reverse?
  (coordinate time, proper time, internal clock time)?
- Quantum Gravity: no time - no time reversal !?
  $$\Psi^{T}[e_{ij},\varphi] = \Psi^{*}[e_{ij},\varphi]?$$
  but this is consistent with the semi-classical approach !

[11] some authors doubt this

The Direction of Time

The following general time asymmetries are observed in our universe[12]:

- The time arrow of radiation
- The thermodynamical arrow of time
- The quantum mechanical arrow of time
- The time arrow of spacetime geometry
- The time of quantum cosmology

You may add: The psychological arrow of time.
But: This is outside the realm of physics

[12] classification according to H.D. Zeh

The arrow(s) of time: questions

- Why does time have an arrow?
- Why do the basic laws of physics do not reveal this arrow?
- Why is the arrow apparent in macro processes but not in micro processes?
- How is the (thermodynamical) arrow connected with entropy?
- Why are there different arrows of time?
- Why do all these arrows point into the same direction?
- Is there a "master" arrow?

P.C.W. Davies: *The Physics of Time Asymmetry* (1974)
H.D. Zeh: *The Physical Basis Of the Direction Of Time* (1989/2007)
S. Savitt (ed.): *Time's Arrows Today: Recent Physical and Philosophical Work on the Direction of Time* (1997)

The arrow(s) of time: some answers

- Time asymmetries could arise from *time-symmetric dynamical laws* solved with *time-asymmetric boundary conditions*.
- hot candidate for the master arrow: cosmological arrow
  hypothesis: the universe started out with a very low entropy
    but why? initial conditions?
    is the direction of time reversed in a recollapsing universe?

<u>some unconventional answers</u>

arrow of time ...

- Carroll/Chen (2004) ... and eternal inflation
- Kiefer (2004, 2012) ... from semiclassical time
- Mersini-Houghton (2012) ... and multiverse
- Barbour/Koslowski/Mercati (2014) ... and shape space
  $\Longrightarrow$ talk by Tim Koslowski
- Rovelli (2015) ... and perspective bias

The Passage of Time

- flow/march/passage of time is an every-day notion; intuitively the past is fixed, the "now" is real, and the future hasn't yet occured
- special relativity suggests a *Block Universe* in which past/present/future coexist[13]

  Flow of time flow seems incompatible with relativity theory.
- distinguish different views
  - time does not flow

    Mc.Taggart, Gödel
  - time does flow but physics is not able to model this flow

    A. Einstein to H. Bergson:

    the time of a psychologist is not the time of physics
  - time does flow but physics alone is not able to model this flow

    D. Dieks, R.T.W. Arthur, O. Pooley, M. Dorato:

    local becoming, no privileged global present
  - physics could/should be able to describe time flow

[13] A. Einstein: Für uns gläubige Physiker hat die Scheidung zwischen Vergangenheit, Gegenwart und Zukunft nur die Bedeutung einer wenn auch hartnäckigen Illusion.

The Passage of Time: give physics a chance

- G.F.R. Ellis: EBU (Evolving Block Universe); e.g. 1407.7243
- J. B.Hartle: "The physics of now"
  "Past, present, and future ... are properties of a specific class of subsystems of the universe that can usefully be called *information gathering and utilizing systems*."
- C. Rovelli: thermal time hypothesis
- D.Lehmkuhl:
  "... the conformal structure of spacetime allows for a clear distinction between past, present, future ..."

S.F. Savitt: "Being and becoming in modern physics", Stanford Encycl.
F. Weinert: *The March of Time* (2013)

Time and Clocks

What is a clock?

Sean Gryb: "It is possible, in principle, to measure time by a clock"[14]

Julian Barbour

..., clocks play a vital role, yet nobody really asks what they are.

Einstein: "Zeit ist das, was man an der Uhr abliest."

- circular definition of time and clock; which came first?
  Galilei discovered that the small oscillations of a pendulum are isochronous by using his pulse as a clock.
  Later, doctors used pendulum clocks to measure the pulse.
- *Newton* about the relation between clock readings and time: we measure only relative evolution between observable quantities of a physical system a_i and a pointer (clockhand position α, say) of a clock: $a_i(\alpha)$

[14] in his talk at this workshop

How microscopic can a clock be?

H. Salecker, E.P. Wigner PR109 (1958)
Quantum Limitations of the Measurement of Space-Time Distances

... deals with the limitations which the quantized nature of microscopic systems imposes on the possibility of measuring distances between space-time events. ... The accuracy of reading a clock with a given mass is considered and examples for microphysical clocks are given...

No clear-cut conclusion was reached by Salecker/Wigner.

A. Frenkel (2005, 2015):
"... their clock[15] can have a microscopic mass and size only if its accuracy is poor, but if the size is macroscopic, a decent accuracy can be achieved even if the mass is microscopic."

[15] referring to one of the example clocks given by Salecker/Wigner

'Quantum clocks'

Various meanings/clocks

Mayato/Alonso/Equsquiza (2002): "Quantum Clocks and Stopwatches"

- Salecker-Wigner clock(s)
- Peres clock
  "... our chief concern ... how much we are perturbing a system by coupling it to a physical clock."
- Larmor clock, Faraday clock, Rabi clock
- (???) 'Compton clock', Müller/Peters/Chu (2010)
  D. Giulini: "... ein Bubenstück aus jüngster Vergangenheit"[16]

[16] ... a knavery from the recent past

Does Time Exist?

Does Time Exist?

Answer by a

- layman: yes of course, it's 17:55
- philosopher: depends on whether you are
  presentist, possibilist, or eternalist
- common or garden physicist:
  yes, look at the symbols t, T, τ in my equations
- quantum gravitist: most probably not ("the end of time")

It is a lousy question anyhow.

which time ?

what is meant by "exist" ?

We also need to distinguish the

measurement of time

perception of time

use of time

ontology of time

The end of time (and its revival)

end of time - different understandings

- physics can/should be formulated without any notion of time
  $\Longrightarrow$ Rovelli: forget time
- time is not a basic notion in physics
  $\Longrightarrow$ Barbour: end of time
- Quantum Gravity is timeless
  $\Longrightarrow$ majority of experts

revival of time - different understandings

- emergence of time (again different meanings; see next slide)
- despite its appearance, the QG Hamiltonian constraint is not only a "gauge" generator, but a 'genuine' Hamiltonian
  $\Longrightarrow$ Gryb/Thébault: time remains
- it doesn't make sense to apply QM to the whole universe
  $\Longrightarrow$ Smolin: time reborn

The Emergence of Time

Emergence in which sense ? Butterfield/Isham (1999)
Which time emerging ?

- Time as arising from timeless theories;
  examples: Newton's time arising from Jacobi principle
- Newton's time arising from "Einstein's time" in the Newtonian limit
- Emergence of semi-classical time
- Emergence and decoherence
  Claus Kiefer: *Quantum Gravity*, Chap. 10.1
- Emergence and quantum entanglement
  Page D.N, Wootters W.K., PR D27 (1983) 2885
- Transition from a discretized time to a continuous time (Oriti, 2014)
- Emergence in the sense of the AdS/CFT duality
- GR as emerging from an effective theory

Quid est ergo tempus?

St. Augustine's question is still not answered

We know for sure that there is not only one notion of time.

Furthermore we can refine the question by positing it
in a 3D mathematical/physical/philosophical grid:

- mathematical: topological/metric properties, coarse grained
- physical: according to relativity principles and
  the specifics from classical mechanic to quantum gravity
- philosophical: time being
  absolute or relational, conventional,
  tensed or non-tesed, existent or irreal