arXiv:1502.00278

discrete time in quantum gravity

Francesca Vidotto

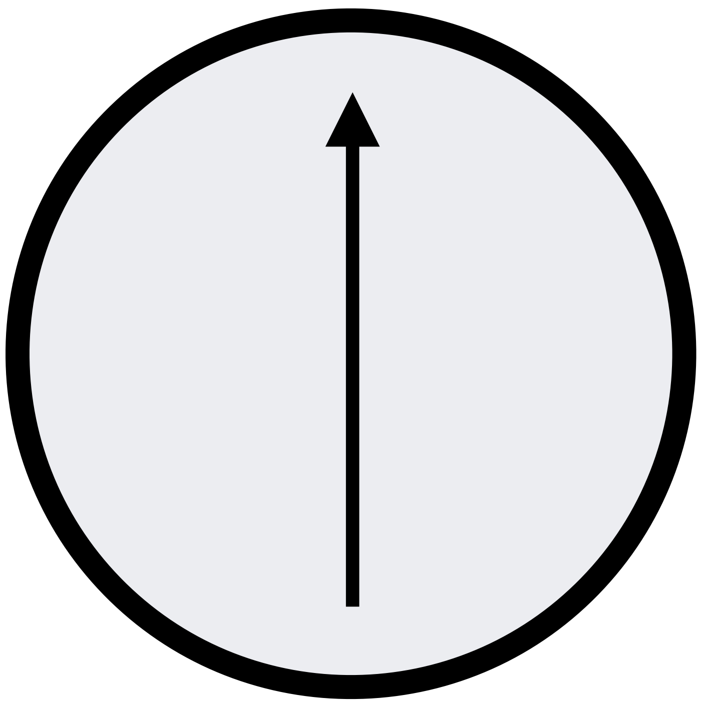
with Carlo Rovelli

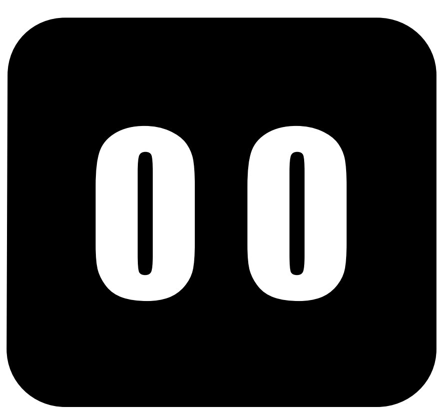
Netherlands Organisation for Scientific Research

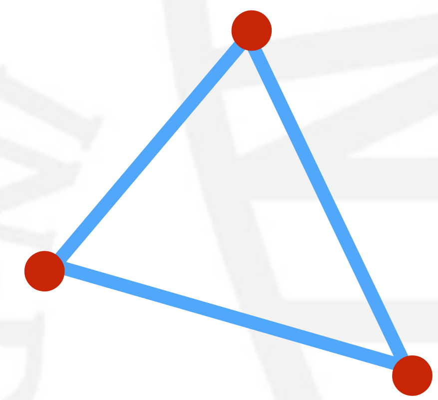
Institute for Mathematics, Astrophysics and Particle Physics

Radboud University Nijmegen

DISCRETE TIME

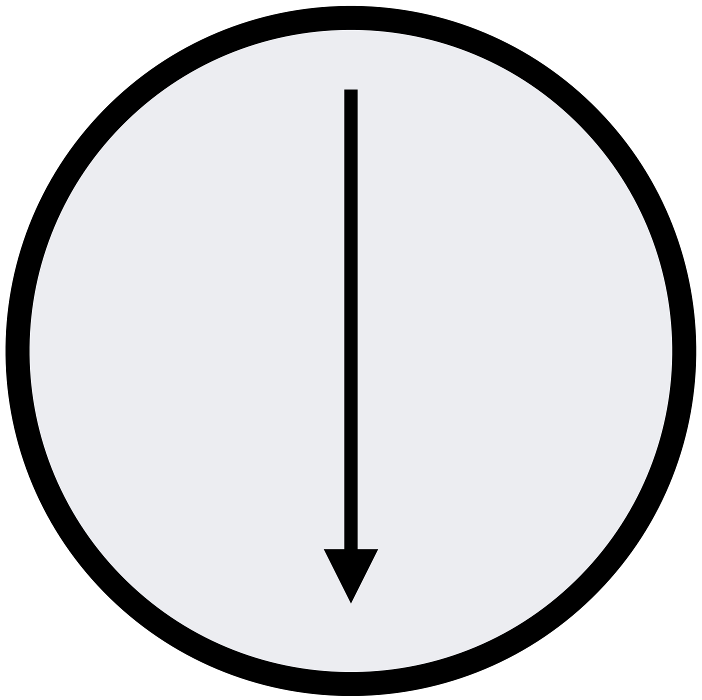


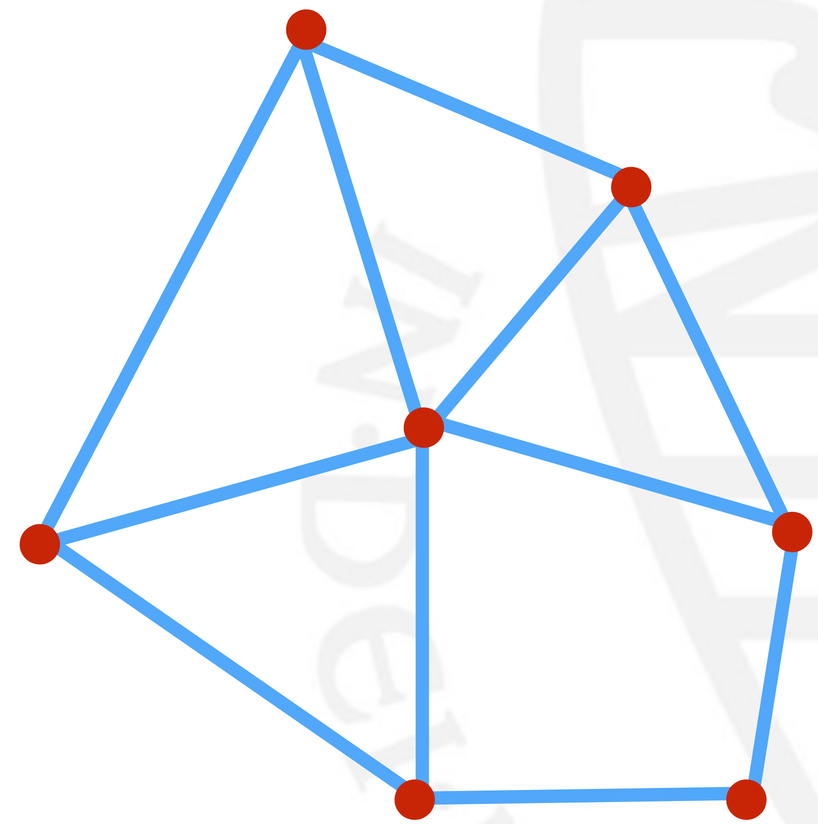




time keeps track of elementary discrete changes

(cfr Heisenberg's S-matrix - Blum's talk)

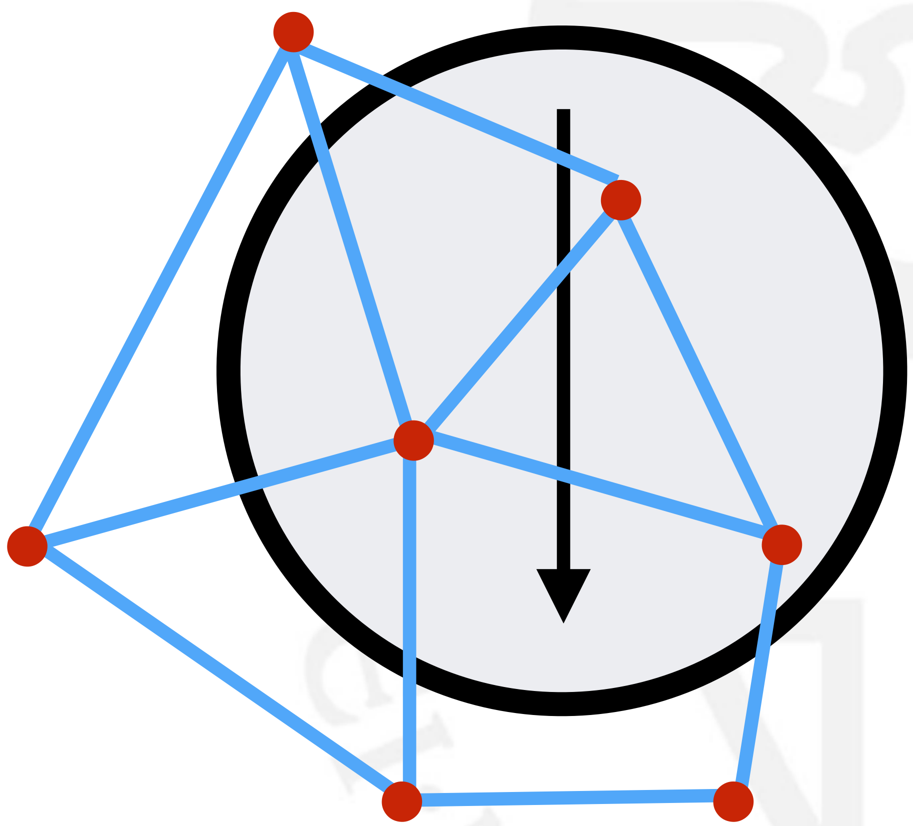




time keeps track of elementary discrete changes

(cfr Heisenberg's S-matrix - Blum's talk)

- Time is just one of the degrees of freedom of the system I am describing.
- Example: cosmology with anisotropies (Bianchi I) $\rightarrow$ I describe one d.o.f. w.r.t. the others!



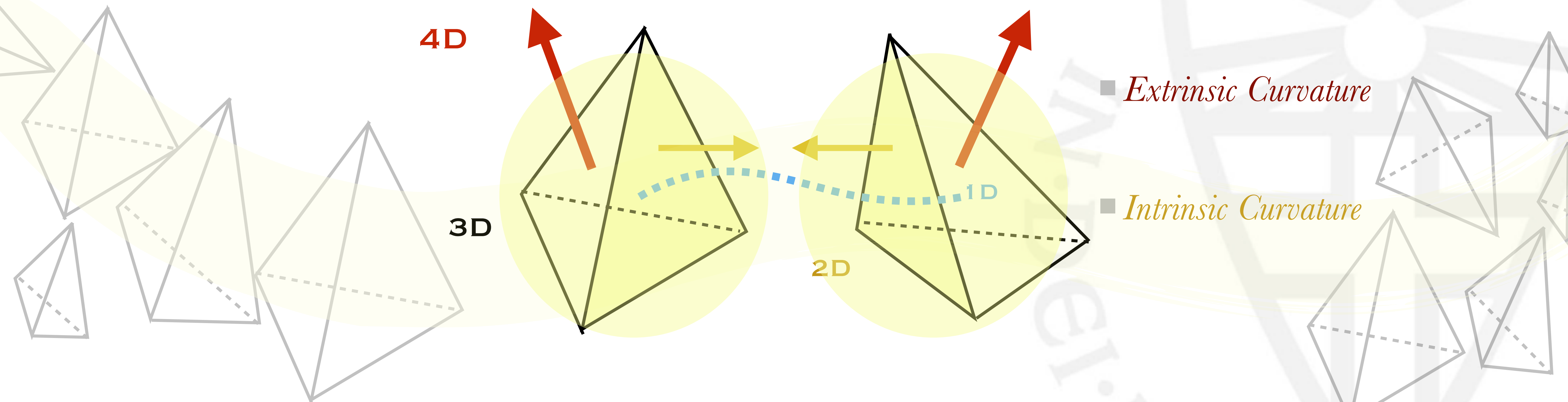
time keeps track of elementary discrete changes

ONTHOLOGY

- SPECIAL RELATIVITY: Space + Time = Spacetime
- QUANTUM FIELD THEORY: Fields have quantum properties
  - DISCRETENESS: Measurements may give discrete spectral values
- GENERAL RELATIVITY: Spacetime is a field $\Longrightarrow$ *general-covariant fields*
  - SUBSTANTIVALISM: Spacetime exists even if there is no Matter
- QUANTUM GRAVITY: Fields have quantum properties $\Longrightarrow$ *general-covariant quantum fields*
  - RELATIONALITY: Observables are relational (QM, GR, gauge theories…)

QUANTUM GEOMETRY

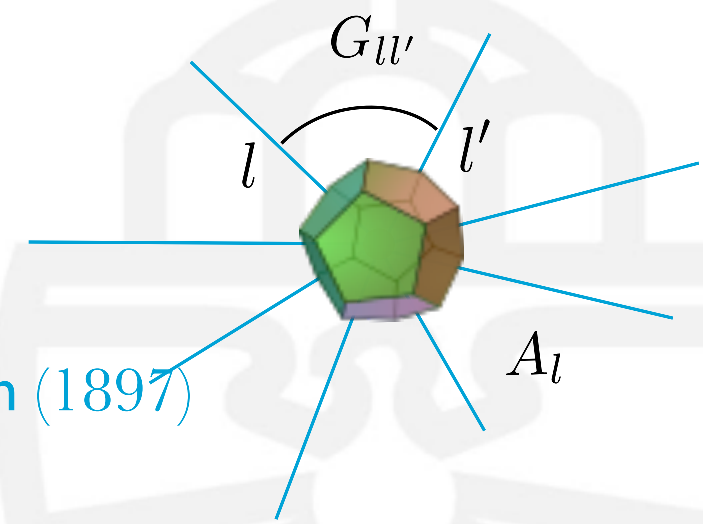
- **Loop Quantum Gravity:**
  - minimal eigenvalues, no minimal "bricks" *(Lorentz invariance)*
  - the operators do not commute
  - quantum superposition



- h_l "Holonomy of the Ashtekar-Barbero connection along the link"
- $\vec{L}_l = \{L_l^i\}, i = 1,2,3$ SU(2) generators $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

  gravitational field operator (tetrad)

- **Achtung:** the length operator is not naturally defined, same for a time operator

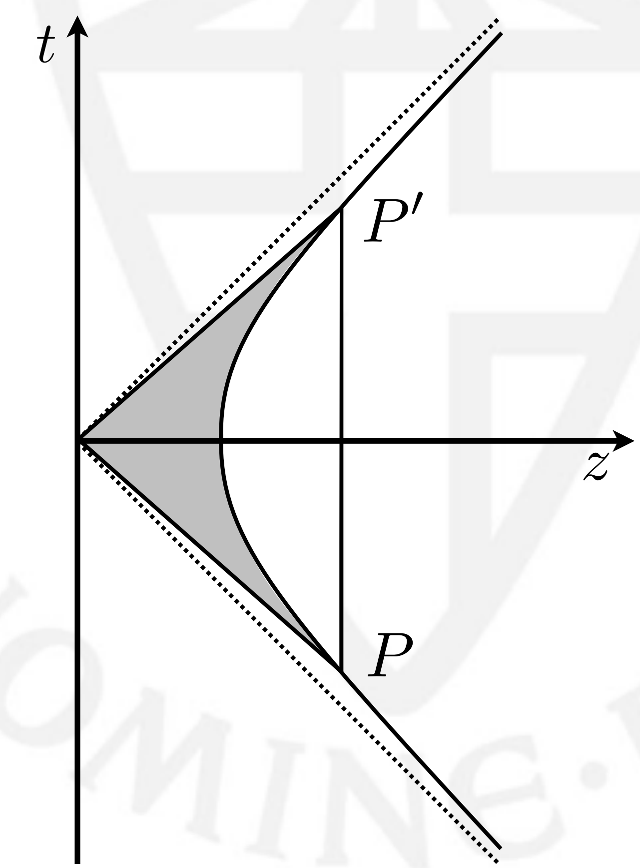


- Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with $\sum_{l \in n} G_{ll'} = 0$

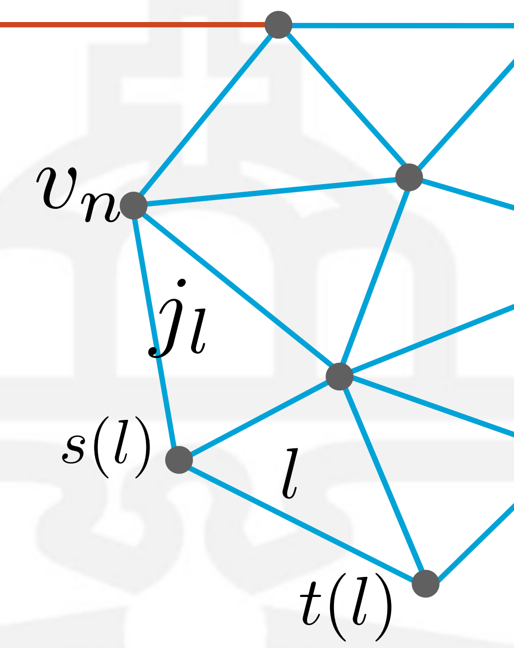
  Penrose's **spin-geometry theorem** (1971), and **Minkowski theorem** (1897)

- Area: $A_\Sigma = \sum_{l \in \Sigma} \sqrt{L_l^i L_l^i}.$

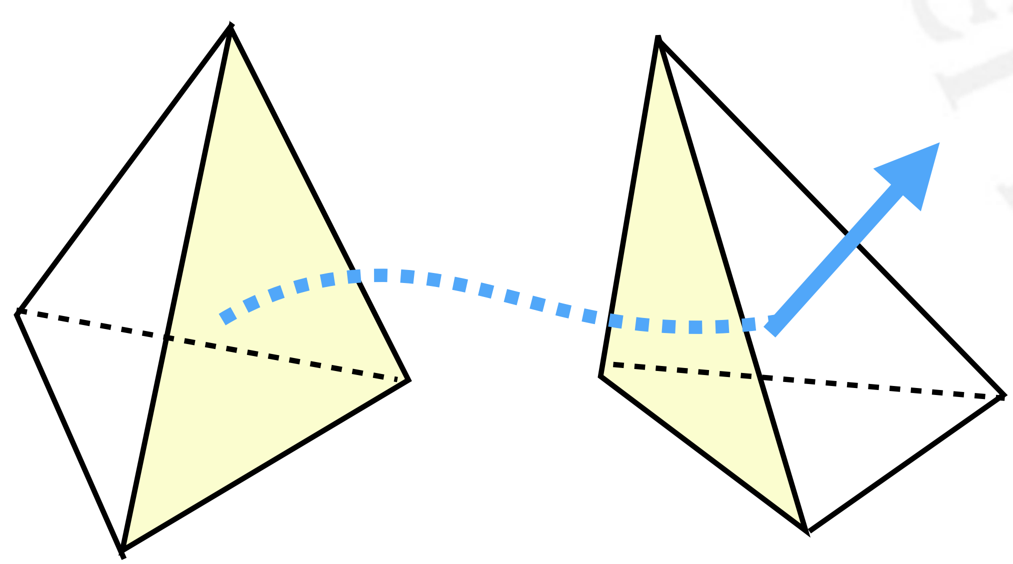
- Lorentzian Area: $A = \int_{\mathcal{R}} e^o \wedge e^i = \int_{\mathcal{R}} \gamma K^i = \int_{\mathcal{R}} L^i$



HILBERT SPACE



- Abstract graphs: $\Gamma=\{N,L\}$
- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$
- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$
- The space $\mathcal{H}_\Gamma$ admits a basis $\Gamma, j_\ell, v_n\rangle$



U(1) x IR

- $h = e^{i\alpha} \in \mathbb{C}, \;\; \beta \in \mathbb{R}$
- Symplectic 2-form: $\omega = d\alpha \wedge d\beta$
- Poisson brakets: $\{h, \beta\} = ih$

- QUANTIZATION

The DISCRETENESS of β is a direct consequence of the fact that α is in a COMPACT DOMAIN.

Notice that $[\alpha, \beta] \neq i\hbar$ because the derivative of the function α on the circle diverges at $\alpha = 0$.

discrete spectrum

Quantization must take into account the global topology of phase space.

The correct elementary operator of this system is not α, but rather $h = e^{i\alpha}$

U(1)xU(1)

- $h = e^{i\alpha}, k = e^{i\beta} \in \mathbb{C}$
- Symplectic 2-form: $\omega = -h^{-1}dh \wedge k^{-1}dk$
- Poisson brakets: $\{k, h\} = hk$

in the limit in which
the radius of one of the two circles can
be considered large we want to recover the
symplectic form of the cotangent space
$\omega = d\alpha \wedge d\beta$

U(1)xU(1)

in the limit in which
the radius of one of the two circles can
be considered large we want to recover the
symplectic form of the cotangent space
$\omega = d\alpha \wedge d\beta$

- $h = e^{i\alpha}, k = e^{i\beta} \in \mathbb{C}$
- Symplectic 2-form: $\omega = -h^{-1}dh \wedge k^{-1}dk$
- Poisson brakets: $\{k, h\} = hk$

QUANTIZATION

- Hilbert space: $|n\rangle \qquad n = 1, ..., N = \dim \mathcal{H}$
- Operators: $k|n\rangle = e^{i\frac{2\pi}{N}n}|n\rangle$

  $h|n\rangle = |n+1\rangle \qquad$ cyclic: $h|N\rangle = |1\rangle$
- Commutator: $[h, k] = \left(e^{i\frac{2\pi}{N}} - 1\right) hk$

  $[\hat{a}, \hat{b}] = i\hbar\widehat{\{a, b\}}$

discrete spectrum

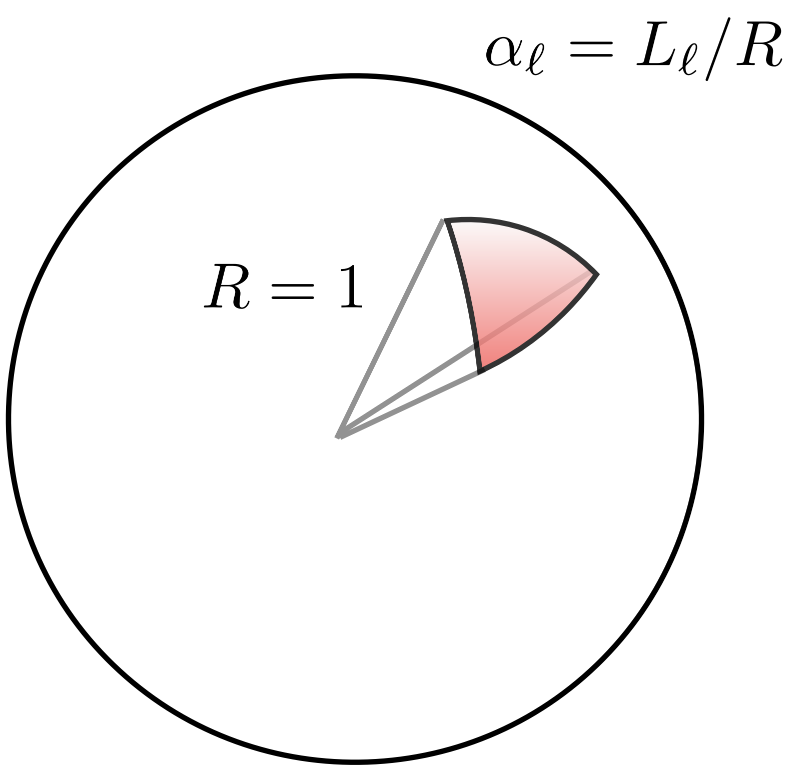
$$\hbar = \frac{2\pi}{N}$$

$$\frac{\text{[Planck length]}}{\text{[cosmological constant]}}$$

IDEA

- Classical kinematics: $\Gamma \equiv su(2) \times SU(2) \quad \forall \ell$
- Idea: replace the algebra with the group —> finiteness (Haggard-Han-Kaminski-Riello '14)
- Classically: compact phase space —> finite Liouville volume
- Quantum: finite # of Planck cells, finite # orthogonal states —> finite dim Hilbert space
- New LQG kinematics (Borissov-Major-Smolin '96, Dupuis-Girelli '13)
- Replacing flat cells with uniformly curved cells (Bahr-Dittrich '09)
- Result: cosmological constant (as the one our universe has !)

CONSTANT CURVATURE GEOMETRY



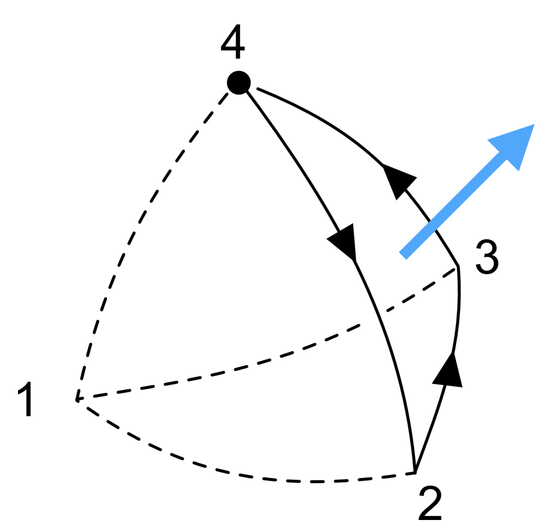
- $\boldsymbol{k}_\ell$: rotation associated to the curved arc ℓ
- $\boldsymbol{h}_\ell$: holonomy of the 3d connection

$$k_\ell, h_\ell \in SO(3) \sim SU(2)$$

- $SU(2) \times SU(2)$ local isometry group, or Chern-Simon gauge group

(Meusburger-Schroers '08)

- small triangles: $k_\ell = e^{\vec{J}_\ell \cdot \vec{\tau}} \sim \mathbb{1} + J_\ell = 1 + \vec{J}_\ell \cdot \vec{\tau}$



- Standard LQG phase space:

$$SU(2) \times SU(2) \underset{R\to\infty}{\longrightarrow} su(2) \times SU(2) = T^*SU(2)$$

$$(k, h) \underset{R\to\infty}{\mapsto} (J, h)$$

limit: arc $\ll$ R
where
$\theta = Tr[kh^{-1}dh]$

- $(k = e^J, h) \in SU(2) \times SU(2)$
- Symplectic 2-form: $\omega = Tr[dk \wedge h^{-1}dh - kh^{-1}dh \wedge h^{-1}dh]$
- … but we are not going to use this!

limit: arc ≪ R
where
$\theta = Tr[kh^{-1}dh]$

- $(k = e^J, h) \in SU(2) \times SU(2)$
- Symplectic 2-form: $\omega = Tr[dk \wedge h^{-1}dh - kh^{-1}dh \wedge h^{-1}dh]$
- … but we are not going to use this!

- QUANTIZATION
  - Hilbert space: $L_2[SU(2)] \sim \oplus_{j=0}^{\infty}(\mathcal{H}_j \otimes \mathcal{H}_j)$
  - Operators: $\langle U|jmn\rangle = D^j_{mn}(U)$
    - $h\psi(U) = U\psi(U)$ $\qquad h_{AB}|jmn\rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}\begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}|j'm'n'\rangle$
    - $J^i\psi(U) = L^i\psi(U)$ $\qquad J^i\,|jmn\rangle = \tau^{i\,(j)}_{mk}\,|jkn\rangle$

limit: arc ≪ R
where
$\theta = Tr[kh^{-1}dh]$

- $(k = e^J, h) \in SU(2) \times SU(2)$
- Symplectic 2-form: $\omega = Tr[dk \wedge h^{-1}dh - kh^{-1}dh \wedge h^{-1}dh]$
- … but we are not going to use this!

- QUANTIZATION QUANTUM GROUPS
  - Hilbert space: $\mathcal{H} = \oplus_{j=0}^{j_{max}} (\mathcal{H}_j \otimes \mathcal{H}_j)$
  - Operators: $\langle U | jmn \rangle = D^j_{mn}(U)$
    - $h\psi(U) = U\psi(U)$ $\quad h_{AB}|jmn\rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}_q \begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}_q |j'm'n'\rangle$
    - $J^i\psi(U) = L^i\psi(U)$ $\quad J^i\,|jmn\rangle = \left(\tau^{i\,(j)}_{mk}\right)_q |jkn\rangle$

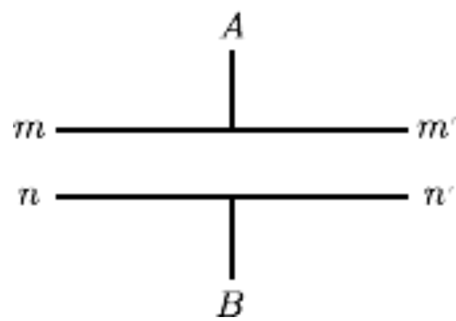
$q^r = -1 \qquad j_{max} = \frac{r-2}{2}$

$$h_{AB}|jmn\rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}_q \begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}_q |j'm'n'\rangle \text{ does not commute any more}$$

$\nexists$ *h reps*

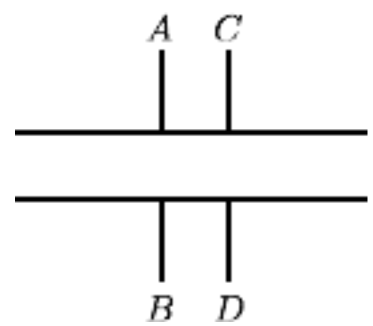
- Wigner symbols as trivalent nodes

$$(h_{AB})^{mn}_{m'n'} \quad =$$



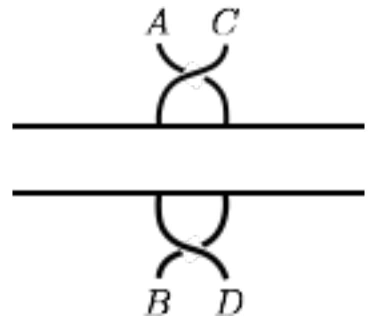
- Acting with two operators:

$$h_{AB}h_{CD} \quad =$$



- Inverse order:

$$h_{CD}h_{AB} \quad =$$



crossing operators

$$h_{AB}h_{CD} = R^{A'C'}_{AC} R^{B'D'}_{BD} \; h_{C'D'}h_{A'B'}$$

Expanding in $\hbar$ so that $R \sim 1 + r$:

$$\{h_{AB}, h_{CD}\} = r^{B'D'}_{BD} h_{CD'}h_{AB'} + r^{A'C'}_{AC} h_{C'D}h_{A'B}$$

quasi Poisson-Lie groups

- **We have introduced a modification of LQG kinematics**
  - compact phase space
  - allows to introduce a (positive) cosmological constant

- **finite dimensional Hilbert space** dim determined by the ratio between the two constants:
  - quantization (physically: Planck constant scale)
  - simplex curvature / deformation of Poisson algebra (physically: cosmological constant)

- Hilbert space reduces to usual LQG one for triangles small compared to curvature radius

- A q-deformation of the dynamics:
  - renders quantum gravity **finite** (Turaev-Viro '92, Han '10)
  - amount to introduce the cosmological constant (Mizoguchi-Tada '91, Han '10)

$$q = e^{i\sqrt{\Lambda}\hbar G}$$

- **Compactness:** discretization of the **intrinsic and extrinsic** geometry

- **Time discreteness:** $K_{ab} \sim dq_{ab}/dt$ where $q_{ab}(\Delta t) \sim q_{ab}(0) + dq_{ab}/dt\ \Delta t$
  minimum proper time Planckian, full discrete spectrum depends on cosmological constant

- Treat space and time on equal foot
- Implement discreteness for all observables
- Treat the variables homogenelly: everything is holonomized
- New: extrinsic geometry turns out to be discrete
- New: time should be discrete too
- This seems to correctly capture the universe as we observe it!