How Theoretical Explorations Explain. A Bayesian Account

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Prologue

Claim

'Minimal' models can explain (Axtell et al. 2002; Cederman 2005; Epstein 2006, chs. 1–2; Tesfatsion 2006).

Objection

How so, if they provide no (empirical) evidence?

- Hausman (1992): models can only *explore* a theory's assumptions, and make more transparent its consequences;
- Guala (2002); Grüne-Yanoff (2009): unlike experiments, models do not confirm (causal) hypotheses

Reply

Casini (2014): minimal models explain when theoretical explorations show what factor makes the explanandum *robust*.

How is this explanatory, exactly?

Prologue

'Theoretical explorations' assess (in)dependence of model results from specific assumptions. When independence obtains, results are *robust*.

'Robustness'

sensitivity analysis: lack of variability of results across parameter values (Saltelli, 2000)

robustness analysis: lack of variability of results across variations in internal structure (Railsback and Grimm, 2011, 302-06)

e.g. probability distribution used to set the parameters, functional forms used to relate the objects' attributes, spatial/relational structure by which objects interact, objects' invoking order and scheduling

network theory: maintenance of network's function across perturbations thanks to network's topology (Rizk et al., 2009)

renormalization group theory (RG): sameness of limiting behavior of systems in a universality class (Lesne and Laguës, 2003, 105-06)

etc.

Prologue

We're interested in what all such techniques have in common wrt explanation.

Two claims

(1) contra (Batterman and Rice, 2014), RG does not provide a general rationalization of how minimal models explain

(2) minimal models explain when the confirmation of an explanatory hypothesis is due to a positive analogy and not to negative analogies — theoretical explorations are meant to show precisely this!

Outline

Outline

1 ABMs of asset pricing

2 Explanation by renormalization?

3 Explanation by robustness analysis

4 Summary

5 Open questions

ABMs of asset pricing

Motivation

- neoclassical economic theory is based on
  - rational expectation hypothesis (REH)

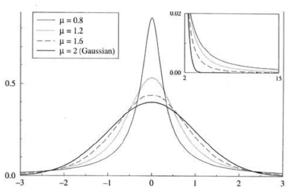
    agents fully rational and informed; maximise expected utility/profits; aggregate behaviour reducible to representative agent's
  - efficient market hypothesis (EMH)

    prices randomly fluctuate around and quickly revert to FVs
- neoclassical economic theory is unable to explain "stylised facts"

(i) fat-tailed distribution of returns

(ii) clustering of volatility

(iii) persistence of volatility



(i)

ABMs of asset pricing

Assumptions

- heterogeneity of individuals

e.g.: (i) phase transition model (Lux and Marchesi, 1999): different *dispositions* (fundamental, optimistic chartist, pessimistic chartist)

e.g.: (ii) evolutionary model (Arthur et al., 1997): different *expectations* (different sets of condition-forecast rules)

- (i) and (ii) having very different auxiliaries

Results

- stylised facts obtain

Explanans

- not external shocks (contra EMH) but an endogenous self-reinforcing process (fundamentalist-chartist switch, use of chartist strategies)
- ultimately, the agents' *heterogeneity* — because the dependence of stylized facts on heterogeneity is *robust*

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Explanation by renormalization?

RG explanation

- what explains sameness of behaviour of large and heterogeneous classes of systems at 'critical' state?
  - e.g. what explains sameness of exponents of scaling laws of fluids and magnets?
- Batterman (2001): asymptotic reasoning in RG

  > *By telling us what (and why) various details are irrelevant for the behavior of interest, this same analysis also identifies those physical properties that are relevant for the universal behavior being investigated (2001, 42)*

  - e.g. spatial dimension of the system, symmetry properties of the explanatory parameter, etc.

Explanation by renormalization?

Critical behaviour is *robust* under perturbations:

> *if one were to alter, even quite considerably, some of the basic features of a system (...) the resulting system (...) will exhibit the same critical behavior (2001, 42)*

- *prima facie* analogous to insensitivity to micro-specifications in ABMs of asset pricing

Batterman and Rice (2014) explicitly argue that RG applies to minimal models outside physics, too

- seems true of at least Lux and Marchesi (1999, 2000)'s phase transition model, inspired by ‘mechanism’ for universality in physics

Explanation by renormalization?

If the analogy were correct, critical behaviour of magnets and markets would be robust *in the same way* — and thus explainable by RG.

But Lux and Marchesi (1999, 2000)'s explanation cannot be so rationalized

Explanation by renormalization?

√ inspired by analogy between phase transitions and stylized facts

> *Statistical physicists have determined that physical systems which consist of a large number of interacting particles obey universal laws that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics. (Stanley et al. 1996, 415; quoted by Lux 2000)*

Explanation by renormalization?

- × makes no use of RG – studies robustness under rescaling *of different objects*, *by different means* and *to different ends*

physics
: uses RG to rescale a *micro* property of the system (viz. particles' couplings) to single out the only relevant parameter at criticality

L&M
: use detrended fluctuation analysis (DFA) (Peng et al. 1994) to rescale a *macro* property of the system – viz. autocorrelation of F(returns) – and show scaling exponent is different from that of autocorrelation of F(FVs), such that the latter does not drive the former

- × even if it did use RG, the result wouldn't be backed up by theory

Explanation by renormalization?

Outside physics, explanation by minimal models rests on:

(i) support by analogies (cf. widespread and casual reference to Bak's SOC);

(ii) theoretical explorations showing this support not to hinge on disanalogies.

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Explanation by robustness analysis

How do theoretical explorations explain?

- they show that the confirmation of an explanatory hypothesis X by the evidence E via analogous factors H is not affected by disanalogies – viz. (unrealistic and possibly false) auxiliaries A
- that is, they show that stylized facts robustly depend on heterogeneity
- to be rationalized in Bayesian terms

***H* systems**

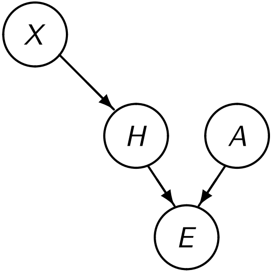
- the more H-systems reproduce E, the larger the confirmation of X
- the more disanalogous the auxiliaries A, the larger the confirmation of X

Non-*H* systems

- the more non-H systems reproduce E, the larger the disconfirmation of X
- the more disanalogous the auxiliaries B, the larger the disconfirmation of X

Explanation by robustness analysis

One system M



variables:

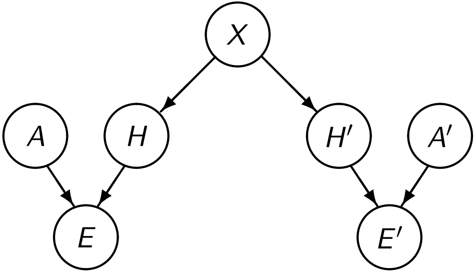
- X: heterogeneity explains (accounts for) stylized facts
- A: auxiliary facts in M obtain
- H: agents in M are heterogeneous
- E: stylized facts obtain in M

$P(X|E) > P(X)$ iff
$P(x) \neq 0$ and $P(h|x) > P(h|\overline{x})$

N.B. equal confirmation of X and (non-modelled) competitors

Explanation by robustness analysis

One analogous system M′



further variables:

A': auxiliary facts in M′ obtain
H': agents in M′ are heterogeneous
E': stylized facts obtain in M′

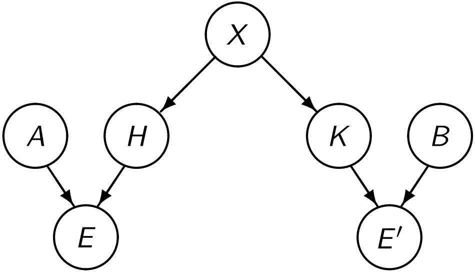
$P(X|EE') > P(X)$ iff

$P(x) \neq 0$
(i) $P(h|x) > P(h|\bar{x})$
(ii) $P(e|ah) > P(e|a\bar{h})$
(iii) $P(e|\bar{a}h) > P(e|\overline{ah})$
(i'-iii') same for M′

N.B. in phase transition model and evolutionary model, $A \perp\!\!\!\perp A'$

Explanation by robustness analysis

One disanalogous system N



further variables:

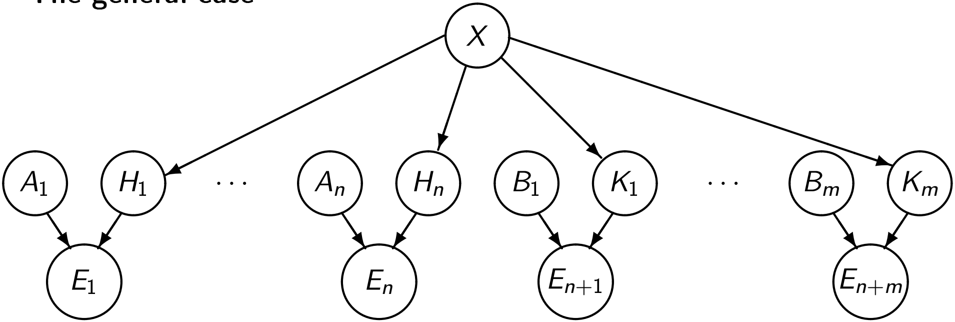
- K: competing explanans in N (alternative to heterogeneity) obtains
- B: auxiliary facts in N obtain
- E': stylized facts obtain in N

$P(X|EE') > P(X)$ iff E confirms X more than E' disconfirms X

e.g. $K=$ agents are homogeneous

Explanation by robustness analysis

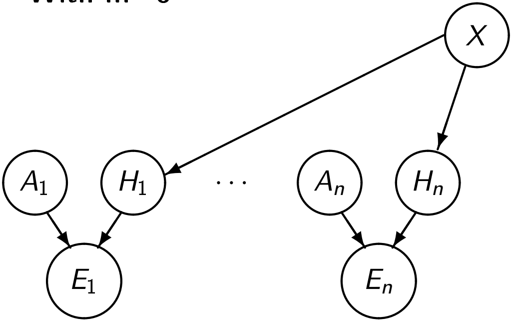
The general case



$$\Delta^{n+m}(X) = P(X|E_1 \ldots E_n E_{n+1} \ldots E_{n+m}) - P(X) \overset{?}{>} 0$$

Explanation by robustness analysis

With m=0



$$\Delta^n(X) = P(X|E_1 \dots E_n) - P(X) > 0 \text{ iff for all } i,\ 2 \leq i \leq n$$

(i) $P(x) \neq 0$

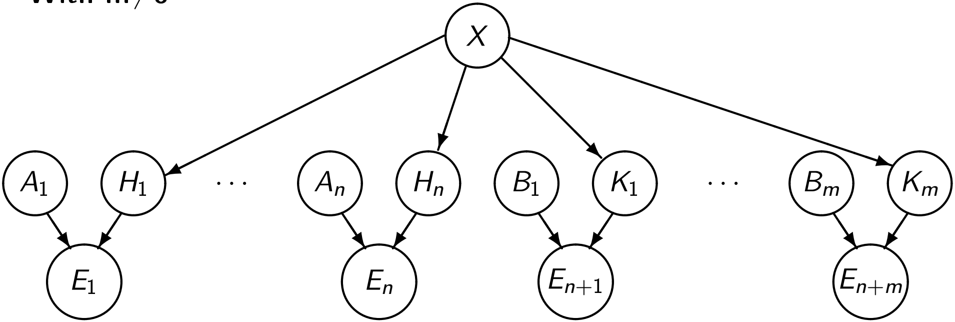
(ii) $P(h_i|x) > P(h_i|\overline{x})$

(iii) $P(e_i|a_i h_i) > P(e_i|a_i \overline{h_i})$

(iv) $P(e_i|\overline{a_i} h_i) > P(e_i|\overline{a_i h_i})$

Explanation by robustness analysis

With m≠0



$\frac{\partial \Delta^{n+m}(X)}{\partial \Phi^n}\Big|_{\text{fixed m}} > 0$ (Δ^{n+m} monot. increasing in n)

$\frac{\partial \Delta^{n+m}(X)}{\partial \chi^m}\Big|_{\text{fixed n}} < 0$ (Δ^{n+m} monot. decreasing in m)

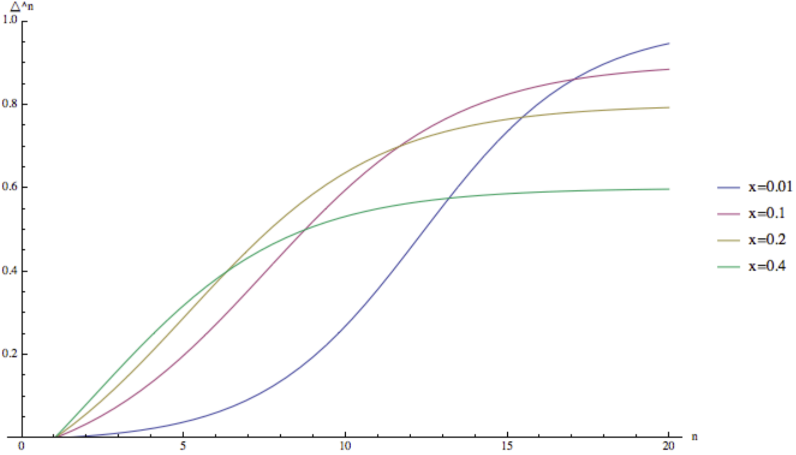
$\lim_{n\to\infty} \Delta^{n+m}(X) = \overline{x}$ (max conf. with $n \to \infty$)

$\lim_{m\to\infty} \Delta^{n+m}(X) = -x$ (max disconf. with $m \to \infty$)

Explanation by robustness analysis

Prior dependence

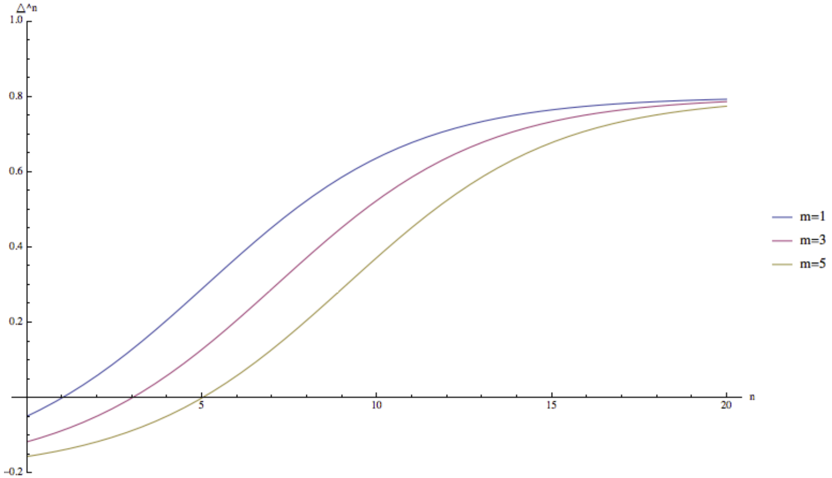
With fixed ratios of H and K



Explanation by robustness analysis

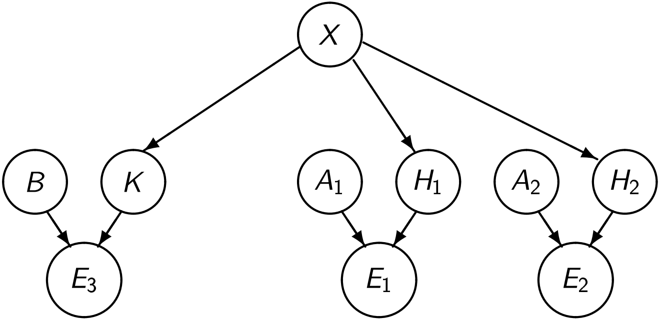
Dependence on number of K systems

With fixed ratios of H and K, prior of X =.2



Explanation by robustness analysis

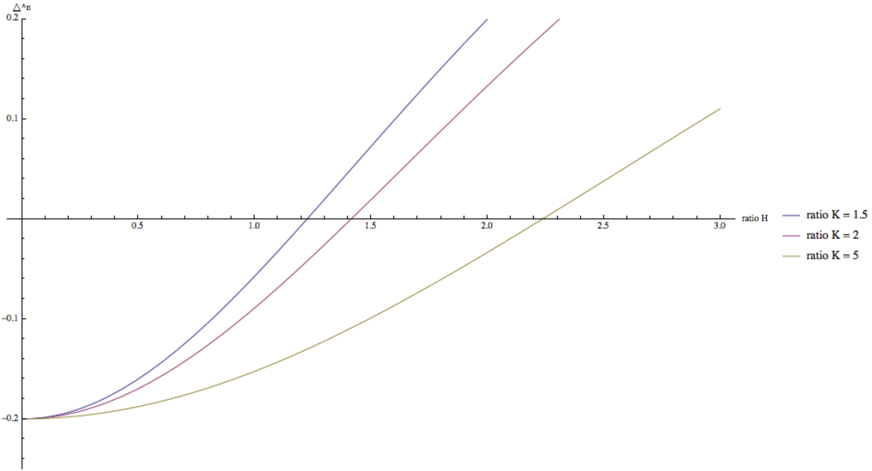
Example: asset pricing $(n = 2,\ m = 1)$



Explanation by robustness analysis

Dependence on ratio of K

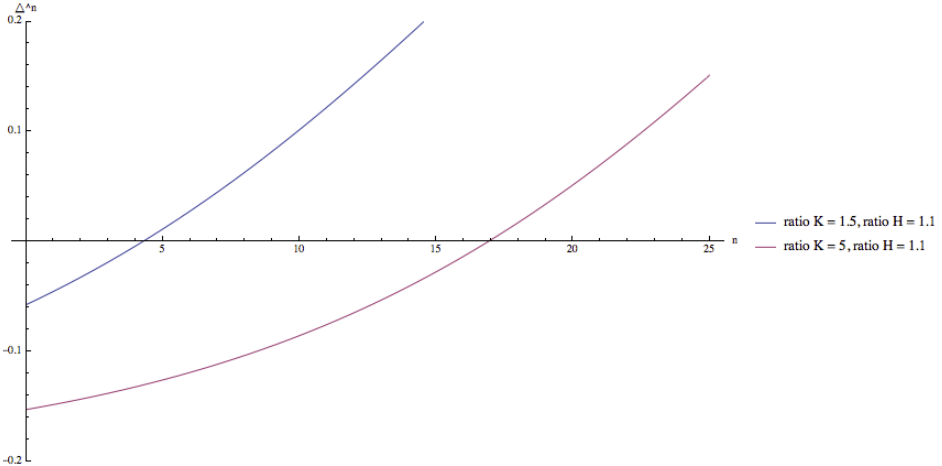
With n=2, m=1, prior of X =.2



Explanation by robustness analysis

Dependence on number of *H*

With n=2, m=1, prior of X =.2



Outline

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Summary

1 Theoretical explorations over 'minimal' models explain — how?

2 RG is not an adequate account outside physics. Between physical and non-physical systems there's a (loose) analogy.

3 Analogical models *can* explain, when confirmation does not hinge on their disanalogies — as shown by theoretical explorations.

Outline

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Open questions

1. Analogies cannot confirm — unless one can prove that artificial and real markets are indeed more analogous than not!
2. Confirmation of X via analogy H is insufficient — aren't there many possible X', X'', etc. confirmed via alternative analogies K, K', etc.?
3. Auxiliaries A, A', A'', etc. are almost never independent — how can this assumption be relaxed?
4. RG is based on robustness, too — can one reconstruct the difference between the two kinds of explanation in this Bayesian framework?

Thank you for listening!

Appendix 1

$$\Delta^{n+m}(X) = P(X|E_1 \dots E_n E_{n+1} \dots E_{n+m}) - P(X) =$$
$$= \frac{P(XE_1 \dots E_{n+m}) - P(X)P(E_1 \dots E_{n+m})}{P(E_1 \dots E_{n+m})}$$

(i) $P(XE_1 \dots E_i \dots E_n \dots E_{n+j} \dots E_{n+m}) = x \prod_{i=1}^{n} \rho^i \prod_{j=n+1}^{m} \sigma^j$

ii) $P(X) = x$

(iii) $P(E_1 \dots E_{n+m}) = x \prod_{i=1}^{n} \rho^i \prod_{j=n+1}^{m} \sigma^j + \overline{x} \prod_{i=1}^{n} \tau^i \prod_{j=n+1}^{m} \upsilon^j$

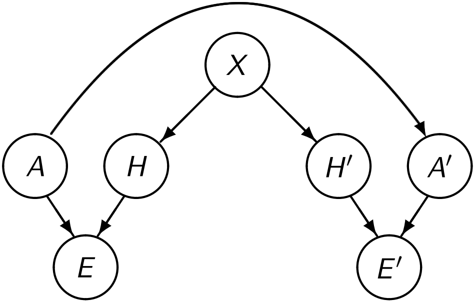
$$\rho^i = h^i_x(a^i e^i_{a^i h^i} + \overline{a^i} e^i_{\overline{a^i h^i}}) + \overline{h^i}_x(a^i e^i_{a^i \overline{h^i}} + \overline{a^i} e^i_{\overline{a^i h^i}})$$
$$\sigma^j = k^j_x(b^j e^j_{b^j k^j} + \overline{b^j} e^j_{\overline{b^j k^j}}) + \overline{k^j}_x(b^j e^j_{b^j \overline{k^j}} + \overline{b^j} e^j_{\overline{b^j k^j}})$$

$$\tau^i = h^i_{\overline{x}}(a^i e^i_{a^i h^i} + \overline{a^i} e^i_{\overline{a^i h^i}}) + \overline{h^i}_{\overline{x}}(a^i e^i_{a^i \overline{h^i}} + \overline{a^i} e^i_{\overline{a^i h^i}})$$
$$\upsilon^j = k^j_{\overline{x}}(b^j e^j_{b^j k^j} + \overline{b^j} e^j_{\overline{b^j k^j}}) + \overline{k^j}_{\overline{x}}(b^j e^j_{b\overline{k^j}} + \overline{b^j} e^j_{\overline{b^j k^j}})$$

$$\Delta^{n+m}(X) = \frac{x\overline{x}\big(\overbrace{\prod_{i=1}^{n} \frac{\rho^i}{\tau^i}}^{\Phi^n} - \overbrace{\prod_{j=1}^{m} \frac{\sigma^j}{\upsilon^j}}^{\chi^m}\big)}{x \underbrace{\prod_{i=1}^{n} \frac{\rho^i}{\tau^i}}_{\Phi^n} + \overline{x} \underbrace{\prod_{j=1}^{m} \frac{\sigma^j}{\upsilon^j}}_{\chi^m}} = \frac{x\overline{x}\,(\Phi^n - \chi^m)}{x\,\Phi^n - \overline{x}\,\chi^m}$$

Appendix 2

Dependence among auxiliaries



e.g. same model with different parameter values (sensitivity analysis)

Appendix 3

Explanation in L&M's model

- Traders
  - fundamentalist: buy (sell) when price is below (above) fundamental value
  - chartist: buy (sell) when optimism (pessimism) prevails
- Pricing
  - calculated by aggregating agents' demands

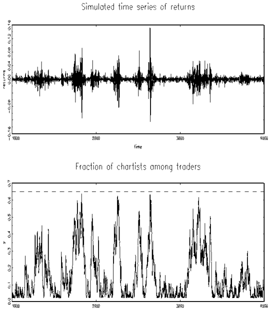
  N.B. FV changes are random
- Switching
  - fundamentalist-chartist switch depends on profit comparison
  - optimist-pessimist switch depends on opinion and price trend

- What happens when (on average) price=FV, as in real markets?

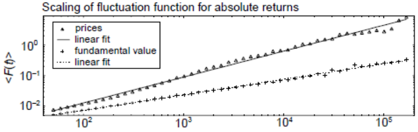
Appendix 3

P1 Equilibrium is unstable (=high volatility).

P2 Volatility correlates with fraction-of-chartists.



P3 Returns and FV changes have different scaling properties (DFA).



C Hence, volatility depends on *switching* — and not on FV changes

N.B. no proof (as per RG) that in real markets the fraction-of-chartist behaviour is described by some critical exponent

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