Single-Domain Free Logic and the Problem of Compositionality

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[1] What is a Single-Domain Semantics for Free Logics?

[1.1] The main components of the semantics

$FM_1 = \{D, I_D(), \mathrm{Den}_D()\}$

D = domain of discourse:

option 1: like in CPL

option 2: **INCLUSIVE**

$\boldsymbol{I_D()}$ = interpretation function: like in CPL

$\boldsymbol{Den_D()}$ = denotation function: **PARTIAL**

$\boldsymbol{\delta_D()}$ = assignment function: like in CPL (**partial, if option 2 (D)**)

$\boldsymbol{V_{M,\,\delta}()}$ = valuation function:

option 1: two-valued like in CPL

option 2: **THREE-VALUED**

option 3: **PARTIAL**

[1.2] Truth-value-conditions for simple sentences

Two-valued semantics for atomic sentences

$V(Pt_1 \ldots t_n) = T$ iff $Den(t_1), \ldots, Den(t_n)$ are all defined and $\langle Den(t_1), \ldots, Den(t_n) \rangle \in I(P)$;

$V(Pt_1 \ldots t_n) = F$ otherwise.

Three-valued/partial semantics for atomic sentences

$V(Pt_1 \ldots t_n) = T$ iff $Den(t_1), \ldots, Den(t_n)$ are all defined and $\langle Den(t_1), \ldots, Den(t_n) \rangle \in I(P)$;

$V(Pt_1 \ldots t_n) = F$ iff $Den(t_1), \ldots, Den(t_n)$ are all defined and $\langle Den(t_1), \ldots, Den(t_n) \rangle \notin I(P)$;

$V(Pt_1 \ldots t_n) =$ **N** /undefined iff It is not the case that $Den(t_1), \ldots, Den(t_n)$ are all defined.

Two-valued/three-valued/partial semantics for singular existential sentences

$V(E!t) = T$ iff $Den(t)$ is defined.

$V(E!t) =$ F / N /undefined iff $Den(t)$ is undefined.

Two-valued semantics for identity sentences:

V(s=t) = T iff Den (s) and Den(t) are both defined and Den(s) = Den(t);

V(s=t) = F otherwise.

Three-valued/partial semantics for identity sentences

V(s=t) = T iff Den(s) and Den(t) are both defined and Den(s) = Den(t);

V(s=t) = F iff Den(s) and Den(t) are both defined and Den(s) ≠ Den(t);

V(s=t) = N / undefined iff It is not the case that Den(s) and Den(t) are both defined.

[2] What are the Main Problems of Compositionality for Single-Domain Semantics?

Problem 1: *The general problem of compositionality with respect to all simple sentences*

Two-/ and three-valued single domain semantics provide a compositional semantics (determination of truth-values) of simple sentences with only **referring** singular terms.

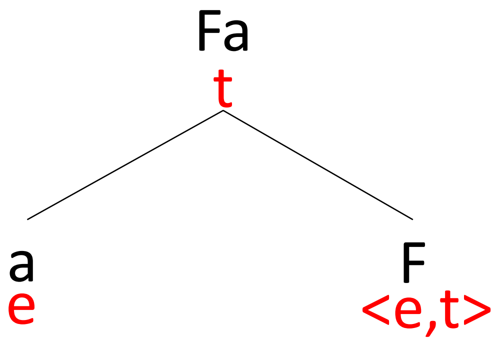
But a non-compositional assignment of truth-values for simple-sentences that contain a **non-referring** term.

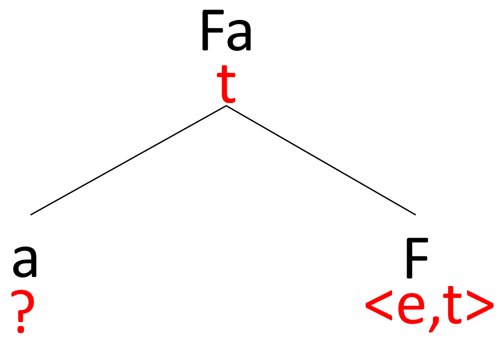
Non-compositional assignments of truth-values are stipulated and seem unprincipled.

A proper formal semantics provides a compositional determination of truth-values for all meaningful and non-defective sentences. (At least such a semantics seems to be preferable philosophically.)

Partial single domain semantics bites the bullet!

Price: a very weak logic.





Problem 2: *A particular problem of compositionality with respect to singular existential sentences*

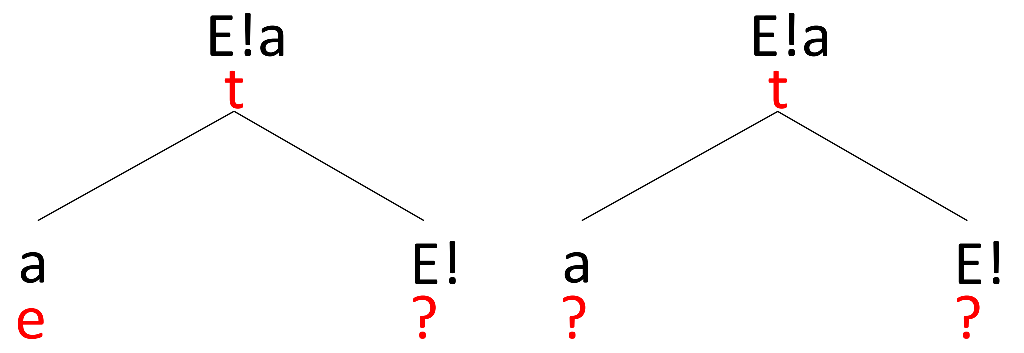
V(E!t) = T iff Den(t) is defined.

V(E!t) = F / N /undefined iff Den(t) is undefined.

Neither in the case of a referring nor a non-referring „t“ does the semantic value of „E!“ play any role concerning the determination of the truth-value of a sentence of the form „E!t“ according to standard (two- and three-valued) single-domain semantics.

The term „E!“ must have a semantic value to be meaningful.

This semantic value should play a significant role with respect to the determination of the truth-value of a sentence of the form „E!t“.



Problem 3: *The problem of a correct compositional semantics with respect to different kinds of **negations** of singular existential sentences*

(1) Sandy Island does **not** exist. (copula negation: $a \notin [E!]$)

(2) Sandy Island is **non-**existent. (predicate negation: $a \in N(E!)$)

(3) It is not the case that Sandy Island exists. (propositional negation: $\neg (a \in [E!])$)

Literal unstressed uses of sentences like (1)-(3) seem to be intuitively **equivalent in truth-value** whether these sentences contain referring or non-refering names although (1)-(3) contain ***compositionally*** different kinds of negation.

Single-domain free semantics has problems to account for this **data** and the **compositional** semantic differences between these constructions.

V(E!t) = F / N /undefined iff Den(t) is undefined.

[3] Towards a New Compositional Single-Domain Semantics for Free Logics

The main idea: We distinguish between the semantic (compositional) value of a singluar term and its semantic referent of a singular term.

The **semantic referent** of a *referring* singular term = **a single object.**

A non-referring singular term does not have a **semantic referent.**

The **semantic value** of a *referring* singular term = **{the semantic referent}**

The **semantic value** of a *non-referring* singular term = **{}**

The denotation function assigns semantic **values** to individual constants, not semantic **referents**.

The First Argument: *The argument from meaningfulness*

In a semantic framework that uses **denotation** and **interpretation** functions as tools to assign meanings/semantic values to non-logical expressions, an expression to which such a function assigns no values is literally *meaningless*.

Sentences that contain meaningless expressions are themselves meaningless and, hence, cannot receive a truth-value as semantic value.

Partial semantics seems the only way to deal with such sentences from a semantic point of view.

Partial semantics is a theoretical option, but, I think, more difficult to justify philosophically and linguistically; especially, because non-referring names are by no means meaningless.

Our distinction between the semantic value and the semantic referent of a singular term allows us also to conceive of non-referring terms as meaningful in our formal semantic framework and, hence, also sentences that contain them as truth-value-apt.

The Main Argument: *The argument from explanatory symmetry*

An **Ockhamist semantics** (**OS**) for monadic atomic sentences makes use of the most basic semantic relations with respect to **names** *and* **predicates**.

(AT) "Fa" is true iff "a" **refers** to o and "F" **applies** to o.

(AF) "Fa" is false iff "a" **refers** to o and "F" does not **apply** to o.

These conditions provide *intuitively plausible* truth- and falsity conditions, at least with respect to the assignment of classical truth-values to atomic sentences. (*If non-referring names are excluded*.)

(AF*) "Fa" is false iff it is not the case that "a" **refers** to o and "F" **applies** to o.

But these conditions are in *no way* **compositional** (in the modern sense of the term).

There are **no** semantic values of predicates that could play this role with respect to (**OS**),

especially if a predicate does not apply to any object. (non-referring terms ≈ non-applying predicates)

That is, I think, the **main reason** why **Fregean** and **Tarskian** semantics do not make us of **(AT)** and **(AF)**.

Tarskian and Fregean semantics assign **abstracted** semantic values, that is: values that are *abstracted away* from the intuitive semantic values (the applicants of a predicate), as semantics values for **predicates** to be able to give compositional truth-conditions for atomic sentences with referring singular terms.

These abstracted values are **(T)** sets of applicants of predicates *or* **(F)** functions from objects into truth-values that pair the applicants of a predicate with the True.

The additional crucial premise for my main argument is:

If Tarskians/Fregeans use abstracted semantic values as semantic values of predicates to be able to account for **compositional** truth conditions for simple sentences that only contain **referring** names,

then it should be allowed for equal reasons to make use of a very similar kind of abstracted semantic values for **names** to account for **compositional** truth conditions for simple sentences with **referring** and **non-referring** names.

[4] A New Compositional Single-Domain Semantics

Compositional two-valued semantics for atomic sentences:

$V(Pt_1 \ldots t_n) = T$ iff $\exists x_1 \ldots . \exists x_n (x_1 \in Den(t_1) \,\&\, \ldots \,\&\, x_n \in Den(t_n) \,\&\, \langle x_1,\ldots,x_n \rangle \in I(P))$;

$V(Pt_1 \ldots t_n) = F$ otherwise.

Compositional three-valued semantics for atomic sentences:

$V(Pt_1 \ldots t_n) = T$ iff $\exists x_1 \ldots . \exists x_n (x_1 \in Den(t_1) \,\&\, \ldots \,\&\, x_n \in Den(t_n) \,\&\, \langle x_1,\ldots,x_n \rangle \in I(P))$;

$V(Pt_1 \ldots t_n) = F$ iff $\exists x_1 \ldots . \exists x_n (x_1 \in Den(t_1) \,\&\, \ldots \,\&\, x_n \in Den(t_n) \,\&\, \langle x_1,\ldots,x_n \rangle \notin I(P))$;

$V(Pt_1 \ldots t_n) = N$ iff It is not the case that $(\exists x_1 \ldots \exists x_n (x_1 \in Den(t_1) \,\&\, \ldots \,\&\, x_n \in Den(t_n)))$.

Compositional two-valued semantics for identity sentences

$V(a=b) = T$ iff $\exists x \exists y(x \in Den(a)\ \&\ y \in Den(b))\ \&\ x=y$.

$V(a=b) = F$ iff $\neg\exists x \exists y(x \in Den(a)\ \&\ y \in Den(b))\ \&\ x=y$.

Compositional three-valued semantics for identity sentences

$V(a=b) = T$ iff $\exists x \exists y(x \in Den(a)\ \&\ y \in Den(b))\ \&\ x=y$.

$V(a=b) = F$ iff $\exists x \exists y(x \in Den(a)\ \&\ y \in Den(b))\ \&\ x \neq y$.

$V(a=b) = N$ iff $\neg\exists x \exists y(x \in Den(a)\ \&\ y \in Den(b))$

[4.1] Towards a Compositional Semantics for Singular Existential Sentences

Compositional first-order semantics for existential sentences

V(E!t) = T iff $\exists x(x \in Den(t) \,\&\, x \in D)$.

V(E!t) = F iff $\neg\exists x(x \in Den(t) \,\&\, x \in D)$.

Compositional second-order semantics for existential sentences

D! = the set of all subsets of D that contain exactly one element.

V(E!*t) = T iff $Den(t) \in D!$

V(E!*t) = F iif $Den(t) \notin D!$

[4.2] The Solution of our Problem 3: an Outline

V(E!*t) = T iff Den(t) $\in$ D!

V(E!*t) = F iif Den(t) $\notin$ D!

Only on this basis **problem 3** can be solved (in a straight-forward way)!

D* = the set of all subsets of D that contain at most one element.

V([t]¬E!*t) = T iff Den(t) $\notin$ D! **(copula negation)**

V([t]N(E!)*t) = T iff Den(t) $\in$ D* – D! **(predicate negation)**

V(¬[t]E!*t) = T iff It is not the case that Den(t) $\in$ D! **(propositional negation)**

[5] A First Extension of Compositional Single-Domain Semantics to Modal Logic

FS_1 = {W, D, D(), I_D(,), Den_D()}

D is the domain of reference

D(w) is the variable domain of quantification

$V_\delta(\forall xA, w) = T$ iff for every object $d \in D(w)$ it is such that $V_{\delta*}(A, w) = T$ where $\delta*$ that is the same as δ except that it assigns {d} to x.

Compositional intensional second-order semantics for singular existential sentences

D!(w) = the set of all subsets of D(w) that contain exactly one element.

$V(E!{*}t, w) = T$ iff $\text{Den}(t) \in D!(w)$

$V(E!{*}t, w) = F$ iff $\text{Den}(t) \notin D!(w)$

[6] A Second Extension of Compositional Single-Domain Semantics to Modal Logic

*Is a **logical existence predicate** relative to **constant domain** semantics for modal logics based on our new framework possible?*

FS_2 = {W, D, D*, I_D(,), Den_D()}   D* is the constant domain of quantification

D* contains individual concepts as elements.

Individual concepts are total functions from worlds to singletons or the empty set

Example 1: The individual concept (Dolf) maps all and only worlds relative to which Dolf exists to the singelton {Dolf}; all other worlds to the empty set

Example 2: The individual concept (Sandy Island) maps all worlds to the empty set.

A new hierarchy of semantic values and referents

The primary **semantic value** of an individual constant = individual concept

The secondary **semantic value** of an individual constant = the set that contains the referent

The **denotation** function assigns **the primary semantic values** to individual constants.

A compositional second-order intensional semantics for existential sentences

D!!(w) = *the set of all values of all individual concepts relative to M and w that contain exactly one element.*

V(E!*t, w) = T iff Den(t)[w] $\in$ D!!(w)

V(E!*t, w) = F iff Den(t)[w] $\notin$ D!!(w)