Bayesian epistemology I: Probabilism and its Dutch Book argument

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We wish to:

(i) present a **model** of an agent's *doxastic* or *credal* states;

(ii) state **norms** that govern those states in terms of our model;

(iii) give **arguments** in favour of those norms.

1 The model

Represent an agent's credal state at a given time t by a *credence function*

$$c_t : \mathcal{F} \to [0, 1].$$

where $\mathcal{F}$ is the algebra of propositions about which the agent has an opinion.[1]

- If $A \in \mathcal{F}$, then $c_t(A) = 0$ iff the agent has minimal credence in A at t.
- If $A \in \mathcal{F}$, then $c_t(A) = 1$ iff the agent has maximal credence in A at t.

Note: It is an empirical assumption that agents are capable of maximal and minimal credences; it is not a normative claim.

2 The norms

There are two types of norms:

- *Synchronic norms* concern the properties of a credence function at a given time.
- *Diachronic norms* concern the relationship between credence functions at different times.

[1]Since $\mathcal{F}$ is an algebra, it is closed under conjunctions, disjunctions, and negations.

2.1 Synchronic norms

At any time t in her epistemic life, an agent ought to have a credence function c_t such that

- **Probabilism** c_t is a probability function.

  That is,

  - $c_t(\bot) = 0$ and $c_t(\top) = 1$.
  - $c_t(A \vee B) = c_t(A) + c_t(B)$ if A and B are mutually exclusive.

- **Countable additivity** c_t is countably additive.

  That is, if $\mathcal{F}$ is infinite,

  - $c_t(\bigcup_n A_n) = \sum_n c_t(A_n)$ if $A_1, A_2, \ldots$ are pairwise mutually exclusive.

- **Regularity** If $A \not\equiv \top$, then $c_0(A) < 1$.

- **Principal Principle** For any probability functon ch,

  $$c_t(A \mid \textit{The ur-chance function is ch}) = ch(A|E_t)$$

  where E_t is the agent's total evidence at t.[2]

- **Reflection Principle** For any $t' > t$,

  $$c_t(A \mid \textit{My credence function at } t' \textit{ is } c_{t'}) = c_{t'}(A)$$

2.2 Diachronic norms

For any two times $t' > t$ in an agent's epistemic life, an agent ought to have credence functions c_t and $c_{t'}$ such that

- **Bayesian Conditionalization** $c_{t'}(A) = c_t(A|E_{t'})$ where $E_{t'}$ is the agent's total evidence at t'.

[2]Conditional probabilities are given by the so-called Ratio Formula:

$$c_t(A|B) = \frac{c_t(A \wedge B)}{c_t(B)}$$